I

$\tan\lambda = \dfrac{4\cos\theta + r'\sin\theta}{-r\sin\theta + r'\cos\theta}$

Vertical Tan when denom $= 0$

$\theta = \{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}\}$

$r = 1+\sin\theta$

$r' = \cos\theta$

$-(1+\sin\theta)\sin\theta + \cos^2\theta \overset{CK}{=} 0$

$-\sin\theta - \sin^2\theta + \cos^2\theta = 0$

$\sin^2\theta + \sin\theta - (1-\sin^2\theta) = 0$

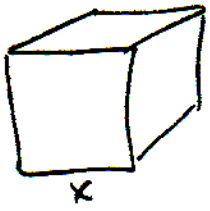
$2\sin^2\theta + \sin\theta - 1 = 0$

$(2\sin\theta - 1)(\sin\theta + 1) = 0$

$\sin\theta = -\frac{1}{2}$ $\quad \sin\theta = 1$

$\frac{7}{2}$ $\quad -1$

II



$\frac{dV}{dt} = K(6S)$ $\quad V = x^3$

$S = x^2$ $\quad \frac{dV}{dt} = 3x^2\frac{dx}{dt}$

$\frac{dx}{dt} = ?$ $\quad 6Kx^2 = 3x^2\frac{dx}{dt}$

$\frac{dx}{dt} = 2K$ since both 2 and K are constants, $\frac{dx}{dt}$ is a constant.

III

A) $y = 3^{\sec x}$

$\ln y = \sec x \ln 3$

$\frac{1}{y}\frac{dy}{dx} = \ln 3 \sec x \tan x$

$\frac{dy}{dx} = 3^{\sec x}(\ln 3)\sec x \tan x$

B) $y = \sin^{-1} e^x$

$\frac{dy}{dx} = \frac{e^x}{\sqrt{1+e^{2x}}}$

C) $y = x^3 \cos^5 7x$

$u \quad v$

$\frac{dy}{dx} = 3x^2\cos^5 7x + 35x^3\cos^4 7x \sin 7x$

IV

$$\overset{u}{2x}\overset{v}{\sin y} + \overset{u}{y}\overset{v}{\cos x} - 2y = 0$$

$$2\sin y + 2x\cos y\frac{dy}{dx} + \cos x\frac{dy}{dx} - y\sin x - 2\frac{dy}{dx} = 0$$

$$2\sin\frac{\pi}{2} + 2\left(\frac{\pi}{2}\right)\cos\frac{\pi}{2}\frac{dy}{dx} + \cos\frac{\pi}{2}\frac{dy}{dx} - \frac{\pi}{2}\sin\frac{\pi}{2} - 2\frac{dy}{dx} = 0$$

$$2 + 0 + 0 - \frac{\pi}{2} - 2\frac{dy}{dx} = 0$$

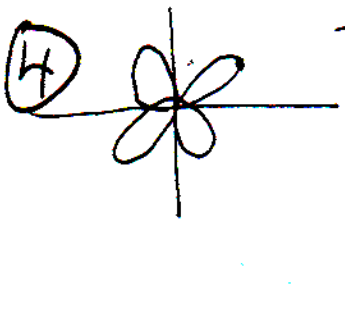
$$\frac{dy}{dx} = 1 - \frac{\pi}{4}$$

V

A) $r = 4\sin 2\theta$

θ	r
0	0
45	4

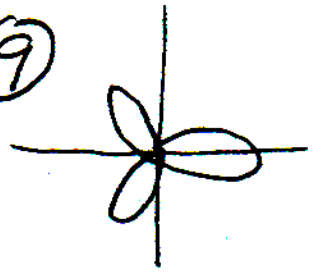
(4)



B) $r = 2\cos 3\theta$

r	θ
2	0
0	30
	15
$-\sqrt{2}$	45

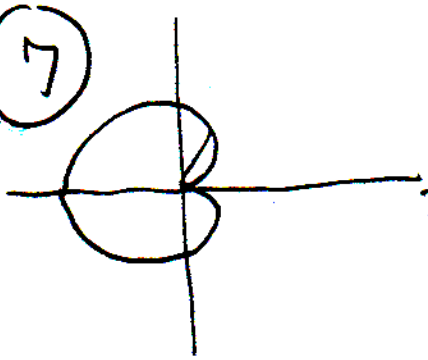
(9)



C) $r = 1 - \cos\theta$

r	θ
0	0
1	90
2	180
$\frac{2-\sqrt{2}}{2}$	45

(7)



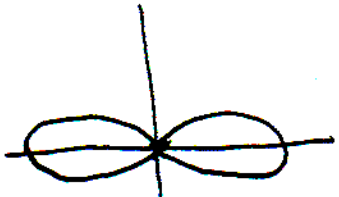
$1 - \frac{\sqrt{2}}{2}$

D) $r^2 = 4\cos 2\theta$

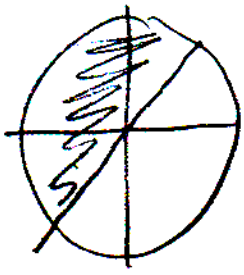
$r = \pm 2\sqrt{\cos 2\theta}$

r	θ
± 2	0
$\pm\sqrt{2}$	30
0	45

(5)



θ	2θ	$\cos 2\theta$	$\sqrt{4\cos 2\theta}$
0	0	1	± 2
30	60	$\frac{1}{2}$	$\pm\sqrt{2}$
45	90	0	0
225	450	0	0
240	480	<0	



A.P. Calculus

Name Calc Chill MU

Copy original problem.
Convince *me* that **you** understand the concept!

Per _____ Date _____

Chapter 11 Make Up Exam

I The angle of inclination λ of the tangent to a curve at the point P(r, θ) on it is given by:

$\tan\lambda = \dfrac{4\cos\theta + r'\sin\theta}{-r\sin\theta + r'\cos\theta}$ where r' means the derivative of r WRT θ. (20 pts)

Determine all θ which would have a vertical tangent to : $r = 1 + \sin\theta$.

II If the rate at which an ice cube melts in cubic inches per second is directly proportional to the surface area with proportionality constant k, show that a single side of the cube decreases at a constant rate. (15 pts)

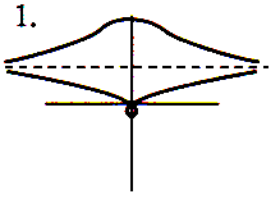
III Given y, find $\dfrac{dy}{dx}$. Final answer must **not** include the letter y. (10 pts ea)

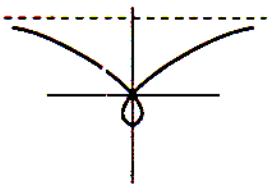
A) $y = 3^{\sec x}$ B) $y = \sin^{-1} e^x$ C) $y = x^3\cos^5 7x$

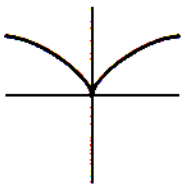
IV Given: $2x\sin y + y\cos x - 2y = 0$. Find $\dfrac{dy}{dx}$ at $\left(\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$. (15 pts)

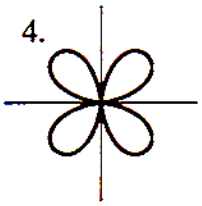
V For each of the following, determine the graph which most closely resembles the given equations. (5 pts ea)

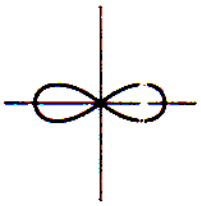
A) $r = 4\sin 2\theta$ B) $r = 2\cos 3\theta$ C) $r = 1 - \cos\theta$ D) $r^2 = 4\cos 2\theta$

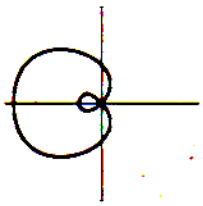
1. 

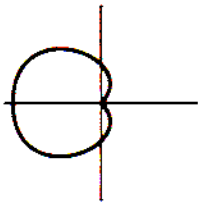
2. 

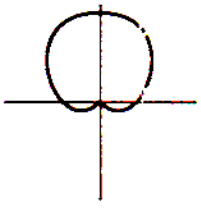
3. 

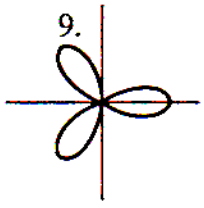
4. 

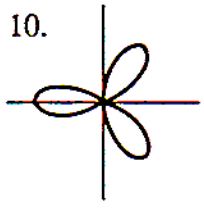
5. 

6. 

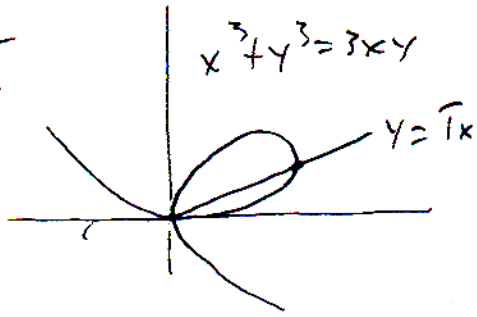
7. 

8. 

9. 

10. 

I



a) $x^3+y^3=3xy$

$x^3+T^3x^3=3x\cdot Tx$

$x+T^3x=3T$

$x=\frac{3T}{1+T^3}$

b) $x^3+y^3=3xy$

$\frac{y^3}{T^3}+y^3=3y\left(\frac{y}{T}\right)$

$y^3\left(\frac{1+T^3}{T^3}\right)=\frac{3y^2}{T}$

$y=\frac{3}{T}\cdot\frac{T^3}{1+T^3}$

$y=\frac{3T^2}{1+T^3}$

c) $T=1 \quad x=\frac{3}{2},\ y=\frac{3}{2}$

$3x^2+3y^2\frac{dy}{dx}=3y+3x\frac{dy}{dx}$

$\frac{dy}{dx}=\frac{y-x^2}{y^2-x} \rightarrow \frac{\frac{3}{2}-\frac{9}{4}}{\frac{9}{4}-\frac{3}{2}}=-1$

$y-\frac{3}{2}=-1\left(x-\frac{3}{2}\right)$

$y=-x+3$

d) $\frac{dy}{dx}=\frac{y-x^2}{y^2-x}=0$ or

$y=x^2$

$x^3+x^6=3x^3$

$x^6-2x^3=0$

$x^3(x^3-2)=0$

$x\in\{0,\sqrt[3]{2}\}$

$y\in\{0,\sqrt[3]{4}\}$

$(0,0)$ is the Node

$\frac{d^2y}{dx^2}=\frac{(y^2-x)\left(\frac{dy}{dx}-2x\right)-(y-x^2)\left(2y\frac{dy}{dx}-1\right)}{(y^2-x)^2}$

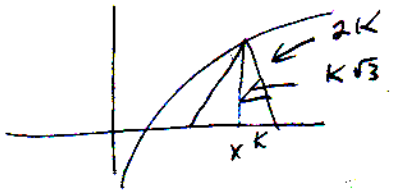
$\frac{\left(2^{\frac{4}{3}}-2^{\frac{1}{3}}\right)\left(0-2\cdot2^{\frac{1}{3}}\right)-\left(2^{\frac{2}{3}}-2^{\frac{2}{3}}\right)\left(2\cdot2^{\frac{2}{3}}\cdot0-1\right)}{(\ \ )^2}$

$2^{\frac{1}{3}}(2-1)\left(-2^{\frac{4}{3}}\right)-0$

<0

$\therefore$ max f'n Test.

II $\frac{dx}{dt}=\sqrt{3}$



a) $\frac{dA}{dt}=?$

$A=\frac{1}{2}bh$

$A=\frac{1}{2}\cdot2K\cdot K\sqrt{3}$

$A=K^2\sqrt{3}$

$A=\frac{(\ln x)^2}{3}\cdot\sqrt{3}$

$A=\frac{(\ln x)^2}{\sqrt{3}}$

$\frac{dA}{dt}=\frac{2}{\sqrt{3}}\cdot\frac{\ln x}{x}\cdot\frac{dx}{dt}$

$=\frac{2}{\sqrt{3}}\cdot\frac{1}{e}\cdot\sqrt{3}=\frac{2}{e}$

$y=K\sqrt{3}=\ln x$

$K=\frac{\ln x}{\sqrt{3}}$

when $y=1$ so

$\ln x=1 \therefore x=e$

Area is increasing at $\frac{2}{e}$ in²/sec.

b) as $x\rightarrow\infty$ $\frac{2}{\sqrt{3}}\cdot\frac{\ln x}{x}\cdot\frac{dx}{dt}$ gets smaller so area continues to increase but at a slower and slower rate.

c) $\left.\frac{dP}{dt}\right|_{x=e}=?$

$P=6K$

$P(x)=6\cdot\frac{\ln x}{\sqrt{3}}$

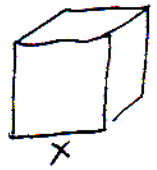
$\frac{dP}{dt}=\frac{6}{\sqrt{3}x}\cdot\frac{dx}{dt}$

$=\frac{6}{\sqrt{3}x}\cdot\sqrt{3}$

$=\frac{6}{e}$

Perimeter grows at $\frac{6}{e}$ in/sec.

III



$\frac{dV}{dt} = k \cdot 6x^2$ $V = x^3$

$\frac{dV}{dt} = 3x^2 \frac{dx}{dt} = k 6x^2$

$\frac{dx}{dt} = ?$

$\frac{dx}{dt} = \frac{6kx^2}{3x^2} = 2k$ constant! The side change is constant.

$A = x^2$

$\frac{dA}{dt} = 2x \frac{dx}{dt}$ → not constant (since it has this x)

IV Polar plot is identical to cartesian Pole = origin

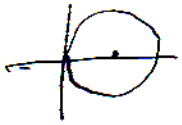
$(\rho, \theta) \to (x, y)$ $x = \rho \cos\theta$ $y = \rho \sin\theta$ $x^2 + y^2 = \rho^2$ $\tan\theta = \frac{y}{x}$

Eccentricity is the ratio of the distance from the focus to a point over the distance from that point to the directrix.

① st line $\theta = \frac{\pi}{4}$ $y = x$

② st line not through Pole $\rho = \frac{1}{\sin\theta}$ $y = 1$

③ Circle center not at pole $\rho = 2\cos\theta$ $(x-1)^2 + y^2 = 1$

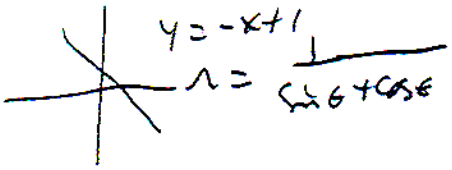


$\rho = \frac{1}{1 + \frac{1}{2}\cos\theta}$ ellipse $e = \frac{1}{2} < 1$

④ Circle center at Pole $\rho = 1$ $x^2 + y^2 = 1$

$\rho = \frac{1}{1 + \frac{3}{2}\cos\theta}$ hyperbola $e = \frac{3}{2} > 1$

⑤ st line Neg Slope $y = -x + 1$

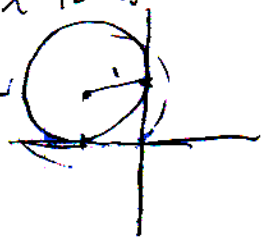


$r = \frac{1}{\sin\theta + \cos\theta}$

⑥ $(x+1)^2 + (y-1)^2 = 1$ $\rho = \frac{1}{1 + \cos\theta}$ Parabola $e = 1$

$x^2 + 2x + 1 + y^2 - 2y + 1 = \rho$ $r^2 + 2r\cos\theta - 2r\sin\theta = -1$

$\rho = \frac{1}{1 + 0\cdot\cos\theta} = 1$ = circle $e = 0$



EXTRA.

If $g'(x) = 2x$ but $g(x) \neq x^2$

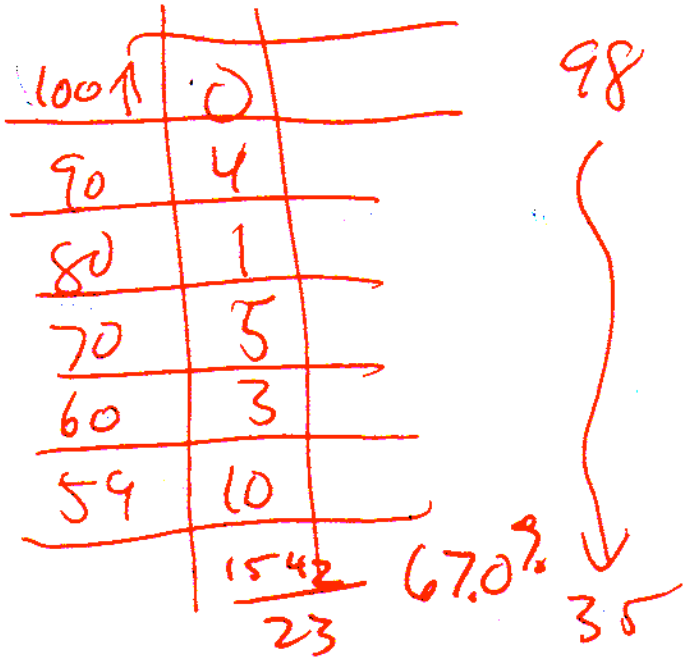
Then $g(x) = x^2 + k$

thus $g(3) - g(1) =$

$3^2 + k - (1^2 + k)$

$9 - 1 = 8$.

Thus, No matter what g is, $g(3) - g(1)$ is still 8.



Excellent Exam

Very unprepared students. They are used to Joking their way through everything this is the 3rd exam this semester with the same results. Get to work or get out. or

Next semester maybe smaller after all.

No changes necessary.

Advanced Placement Calculus

Name Calc Ch 11

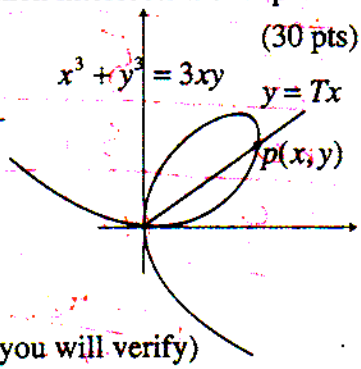
Per _____ Date 2000

Copy original problem.

Convince *me* that **you** understand the concept!

Chapter 11 Exam

I Given the graph of the folium of Decartes. A folium is a plane cubic curve having a single loop, a node, and two ends asymptotic to the same line. We have drawn the line $y = Tx$ which intersects the loop as shown at point P which has coordinates (x, y). (30 pts)



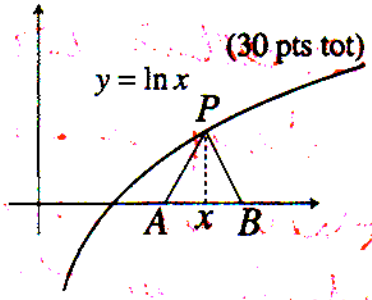
a) Write x in terms of T. Explain the steps necessary to arrive at $x = \dfrac{3T}{1+T^3}$

b) Write y in terms of T .

c) Find the equation of the tangent line when $T = 1$.

6 pts d) Find the coordinates of that point of the loop which is highest. (of course, you will verify)

II A point P with coordinates (x, y) moves along the curve $y = \ln x$ so that its abscissa value (that is the x-coordinate) increases at the rate of $\sqrt{3}$ inches per second. P is one vertex of an equilateral triangle whose other two vertices on the x-axis. (hint: let the line AB have length $2k$.) (30 pts tot)



a) How fast is the area of triangle APB changing when the ordinate of P (the y-coordinate) value is one?

b) As x continues to get larger, what can be said about the rate at which the area of the triangle is growing?

c) How fast is the perimeter of triangle APB changing when the ordinate of P (the y-coordinate) value is one?

III If the rate at which an ice cube melts (in cubic inches per second) is directly proportional to the total surface area of the cube with proportionality constant k, show that a single edge of the cube decreases at a constant rate and that the area of a side does ***not*** change at a constant rate. (20 pts)

IV Describe the similarities and differences between the Cartesian and the polar graphing planes. Include in your discussion directions for converting from one form to the other and back again. List, identify and label with the Cartesian as well as the polar equations: one for a straight line through the pole with positive slope, one for a straight line through the pole with negative slope, one for a straight line with positive slope NOT through the pole; one for a straight line with negative slope NOT through the pole; one for a circle with center at the pole; one for a circle with center NOT at the pole; one for a circle with center located in what would be called the "second quadrant" if we were using Cartesian rather than polar labeling. Briefly describe eccentricity including examples of the equations in polar form of an ellipse, a circle, a hyperbola, and a parabola. (20 pts)

Extra Credit ---------- 5 pts .....

We all know that if $f(x) = x^2$ then $f'(x) = 2x$. If someone knows a function $g(x)$ such that $g'(x) = 2x$ but $g(x) \neq x^2$, what can be said about the difference $g(3) - g(1)$? Explain, of course.

I A) $\tan\theta_1 = \frac{y}{x} = \frac{1}{1} \therefore \theta_1 = 45°$

$\tan\theta_2 = \frac{y}{x} = \frac{25}{5} = 5$

$\therefore \theta_2 = \tan^{-1} 5$

$\theta_2 \approx 78°41'$

B) $\tan\theta = \frac{y}{x} = \frac{x^2}{x} = x$

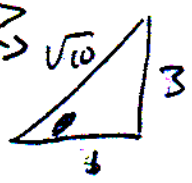
so $x = \tan\theta$ and

since $y = x^2$, $y = \tan^2\theta$.

C)

$\left|\frac{ds}{dt}\right| = \sqrt{\dot{x}^2 + \dot{y}^2}$

$\theta = \tan^{-1} 3 \quad \therefore \tan 3 = \theta$



$\frac{d\theta}{dt} = 2\pi$

$\dot{x} = \sec^2\theta \frac{d\theta}{dt}$

$\dot{y} = 2\tan\theta \sec^2\theta \frac{d\theta}{dt}$

$\dot{x} = 10 \cdot 2\pi = 20\pi$

$\dot{y} = 2 \cdot 3 \cdot 10 \cdot 2\pi = 120\pi$

$\frac{ds}{dt} = \sqrt{(20\pi)^2 + (120\pi)^2}$

$(20\pi \cdot 6)^2$

$= 20\pi\sqrt{1 + 36}$

$= 20\pi\sqrt{37}$

II A) $y = Ae^{kt}$ ① $y = 10 \cdot 2^t$

$20 = 10e^k$ ② $30 = 10 \cdot 2^t$

$\therefore e^k = 2$ $3 = 2^t$

$\ln 3 = t \ln 2$

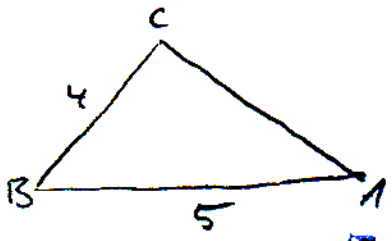
$\frac{\ln 3}{\ln 2} = t$

1.585

1 hr 35 min.

At 4:05 the culture will be triple what it was at 2:30.

II B)



$\frac{dA}{dt} = ?$

$\frac{d\theta}{dt} = .06$

$A = \frac{1}{2} \cdot 4 \cdot 5 \sin\theta$

$A = 10\sin\theta$

$\frac{dA}{dt} = 10\cos\theta \frac{d\theta}{dt}$

$= 10 \cdot \frac{1}{2} \cdot \frac{3}{10}$

$\frac{dA}{dt} = \frac{3}{20}$

The area is increasing at .3 radians per second.

The area will continue to increase until $\theta > 90$ and $\cos\theta < 0$ at which time the area decreases until $\theta = 180°$ and the triangle ceases to exist.

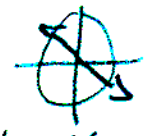
III Polar Plot is identical to Cartesian plot Pole = origin

$(\rho, \theta) \to (x, y)$ $x = \rho\cos\theta$, $y = \rho\sin\theta$, $\frac{y}{x} = \tan\theta$, $x^2 + y^2 = \rho^2$

① ST line through Pole Positive Slope

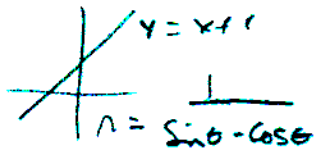
$y = x$, $\theta = \frac{\pi}{4}$

② ST line through pole $M < 0$



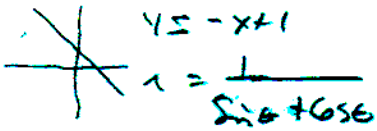
$y = -x$, $\theta = \frac{3\pi}{4}$

③ ST line $m \neq 0$ not through Pole



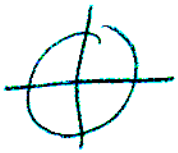
$y = x + 1$, $r = \frac{1}{\sin\theta - \cos\theta}$

④ ST line $m < 0$ No through Pole

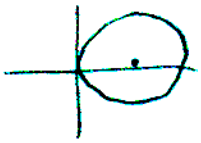


$y = -x + 1$, $r = \frac{1}{\sin\theta + \cos\theta}$

⑤ circle cuts at Pole $r = 1$, $x^2 + y^2 = 1$



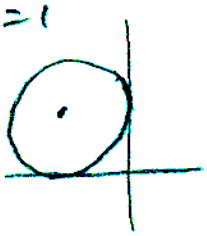
⑥ Circle, cuts not at Pole $r = 2\cos\theta$, $(x-1)^2 + y^2 = 1$



⑦ circle cuts in 2nd Quad $(x+1)^2 + (y-1)^2 = 1$

$x^2 + 2x + 1 + y^2 - 2y + 1 = 1$

$r^2 + 2r\cos\theta - 2r\sin\theta = -1$



Eccentricity is the ratio of the distance from a focus to a point over the distance from that point to the directrix

$\rho = \frac{1}{1 + \frac{1}{2}\cos\theta}$ ellipse $e = \frac{1}{2} < 1$

$\rho = \frac{1}{1 + \frac{3}{2}\cos\theta}$ hyperbola $e = \frac{3}{2} > 1$

$\rho = \frac{1}{1 + \cos\theta}$ Parabola $e = 1$

$\rho = \frac{1}{1 + 0\cos\theta} = 1$ circle $e = 0$

} If a focus is at the pole.

Ext Vol. $V = \pi r^2 h$

$r = 1$

$h = \frac{3}{2}$ inch

$h = \frac{3}{2} \cdot \frac{1}{12} = \frac{1}{8}$ ft.

$V = \pi \cdot 1^2 \cdot \frac{1}{8}$

so volume of stone is $\frac{\pi}{8}$ cu feet.

$V = \pi(1)^2 h$ without stone

$= \pi h$

$V + \text{stone} = \pi(h + \frac{1}{8})$

$= \pi h + \frac{\pi}{8}$

$\therefore$ stone has volume $\frac{\pi}{8}$.

$\frac{3}{2}$ inch $= \frac{3}{2} \cdot \frac{1}{12} = \frac{1}{8}$ ft

$\frac{1}{8}$ ft

678.6 cu in.

Volume of raised part 216π.

Volume of raised part $\pi r^2 \cdot h = \pi \cdot 1^2 \cdot \frac{1}{8} = \frac{\pi}{8}$

Advanced Placement Calculus

Name Calc Ch 11

Copy original problem.

Per _____ Date 5/12/2001

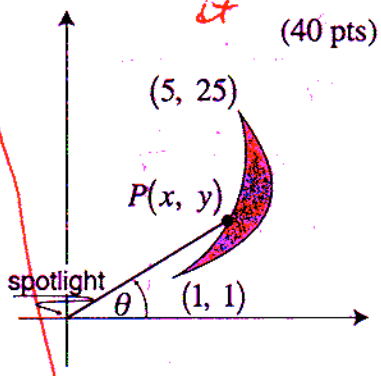
Convince *me* that **you** understand the concept!

* and $\dot{x}$ means $\frac{dx}{dt}$ and $\dot{y}$ means $\frac{dy}{dt}$

98 mu

Chapter 11 etc Exam

I The figure on the right shows a spotlight shining on point $P(x,y)$ on the shoreline of Crescent Island. The spotlight is located at the origin and is rotating. The portion of the shoreline on which the spotlight shines is in the shape of the parabola $y = x^2$ from the point (1, 1) to the point (5, 25). Let θ be the angle between the beam of light and the positive x-axis. (hint: soh-cah-toa). (40 pts)



a) For what values of θ between 0 and 90 degrees does the spotlight shine on the shoreline?

b) Explain how we know the (x,y) coordinates of point P in terms of θ are given by: $(\tan\theta, \tan^2\theta)$.

c) The spotlight is rotating at the rate of one revolution per minute. We can define "speed along a curve" as: $speed = \frac{ds}{dt} = \sqrt{(\dot{x})^2 + (\dot{y})^2}$ (Notice that *speed* is a change wrt *time*.). How fast is the point P traveling along the shoreline at the instant when it is at the point (3,9)? **Be exact. No decimals.**

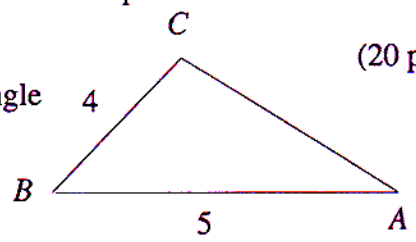
II Related Rates. Answer using a complete, meaningful sentence. (Properly interpreted, of course.)

a) Suppose that a bacterial culture grows at a rate proportional to the amount present. At 2:30 P.M. there are ten grams of the culture and at 3:30 P.M. there are twenty grams.. (total 20 pts)

1) Derive the formula for the amount y of culture present in terms of time, t. (i.e. eliminate c).

2) What time (to the nearest minute) will it be when the culture triples its mass?

b) Given triangle ABC as shown on the right. The angle B is increasing at .06 radians/second. How fast is the area of the triangle changing when angle B is 60 degrees? Is the area increasing or decreasing. Will it continue to increase (or decrease) or when will it change? (20 pts)



III Describe the similarities and differences between the Cartesian and the polar graphing planes. Include in your discussion directions for converting from one form to the other and back again. List, identify and label with the cartesian as well as the polar equations: one for a straight line through the pole with positive slope, one for a straight line through the pole with negative slope, one for a straight line with positive slope NOT through the pole; one for a straight line with negative slope NOT through the pole; one for a circle with center at the pole; one for a circle with center NOT at the pole; one for a circle with center located in what would be called the "second quadrant" if we were using cartesian rather than polar labeling. Briefly describe eccentricity including examples of the equations in polar form of an ellipse, a circle, a hyperbola, and a parabola. (20 pts)

Extra Credit ---------- 5 pts ----------

A stone is dropped into a cylindrical water tank which is two feet in diameter causing the water level to rise one and one-half inches. What is the exact volume of the stone? $V_{cylinder} = \pi r^2 h$

I

a) $\tan\theta_1 = \frac{1}{1} \Rightarrow \theta = \frac{\pi}{4}$

$\tan\theta_2 = \frac{25}{5} = 5 \Rightarrow \theta = \tan^{-1}5$

The lighthouse on the shoreline when $\theta \in [\frac{\pi}{4}, \tan^{-1}5]$

b) $\tan\theta = \frac{y}{x} = \frac{x^2}{x} = x$

so ...

$x = \tan\theta$

$y = \tan^2\theta$

c) $\left|\frac{ds}{dt}\right| = \sqrt{\dot{x}^2 + \dot{y}^2}$

$\theta = \tan^{-1}\left(\frac{9}{3}\right) = \tan^{-1}(3)$

$\frac{d\theta}{dt} = 2\pi$

$\dot{x} = \sec^2\theta \frac{d\theta}{dt}$

$\dot{y} = 2\tan\theta\sec^2\theta \frac{d\theta}{dt}$

$\dot{x} = 10 \cdot 2\pi$

$\dot{y} = 2 \cdot 3 \cdot 10 \cdot 2\pi$

$\frac{ds}{dt} = \sqrt{(20\pi)^2 + (120\pi)^2}$

$= 20\pi\sqrt{1+36}$

$= 20\pi\sqrt{37}$

II

a) $20 = 10Ae^k$

$e^k = 2$

$y = 10(2)^t$

$30 = 10(2)^t$

$3 = 2^t$

$\ln 3 = t \ln 2$

$\frac{\ln 3}{\ln 2} = t$

$t = 1.585$

1 hr 35 min.

at 4:05 pm the culture will be triple of what it was at 2:30

b) $y = Ae^{kt}$

$\frac{1}{2}A = Ae^{1690k}$

$\frac{1}{2} = e^{-1690k}$

$2^{\frac{-1}{1690}} = e^{-1k}$

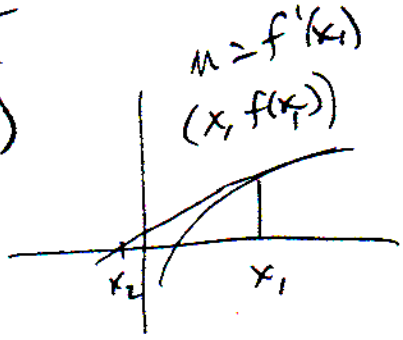
$y = 60(2)^{\frac{-t}{1690}}$

$= 60(2)^{\frac{-100}{1690}}$

$y = 57.59$ gms

I

c)



$y - f(x_1) = f'(x_1)(x - x_1)$

0 ↑ x_2

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$x_{N+1} = x_N - \frac{f(x_N)}{f'(x_N)}$

$f(x) = x^4 + x - 1$

$f'(x) = 4x^3 + 1$

$x_{N+1} = x_N + \frac{x^4 + x - 1}{4x^3 + 1}$

$x_1 = .5$

$x_2 = .737$

$x_3 = .7392$

$x_4 = .739085$

$Y_1 = f(x_1)$

$Y_2 = f(x)$

$Y_3 = x - Y_1/Y_2$

II

D

$\frac{dA}{dt} = ?$

$\frac{d\theta}{dt} = .06$

$A = \frac{1}{2} 4 \cdot 5 \sin\theta$

$A = 10 \sin\theta$

$\frac{dA}{dt} = 10 \cos\theta \frac{d\theta}{dt}$

$= .6 \cos\theta \Big|_{\theta = \frac{\pi}{3}}$

$= .3$

The area will continue to increase until $\theta = \frac{\pi}{2}$.

for $\theta \in [\frac{\pi}{3}, \pi]$

$\frac{dA}{dt}$ will be > 0, increasing and decreasing.

The area is increasing at .3 radians per second

Advanced Placement Calculus

Copy original problem.

Convince *me* that **you** understand the concept!

Name Ch 11 My Key

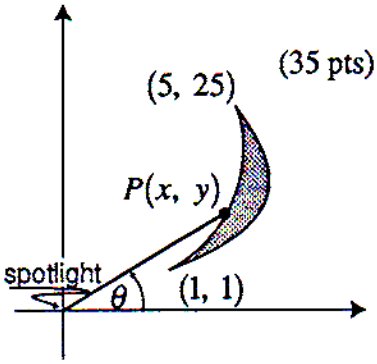
Per _____ Date _____

Calculus 98

Chapter 11 etc Make Up Exam

I

The figure on the right shows a spotlight shining on point $P(x, y)$ on the shoreline of Crescent Island. The spotlight is located at the origin and is rotating. The portion of the shoreline on which the spotlight shines is in the shape of the parabola $y = x^2$ from the point (1, 1) to the point (5, 25). Let θ be the angle between the beam of light and the positive x-axis. (~~No calculators on this problem.~~) (35 pts)



a) For what values of θ between 0 and 2π does the spotlight shine on the shoreline?

b) Find the x- and y- coordinates of point P in terms of θ.

c) The spotlight is rotating at the rate of one revolution per minute. We can define "speed along a curve" as: $speed = \frac{ds}{dt} = \sqrt{(\dot{x})^2 + (\dot{y})^2}$ (Notice that *speed* is a change wrt *time*.). How fast is the point P traveling along the shoreline at the instant when it is at the point (3,9)? No Decimal Answers. Be Exact.

II

Potpourri

a) Suppose that a bacterial culture grows at a rate proportional to the amount present. At 2:30 P.M. there are ten grams of the culture and at 3:30 P.M. there are twenty grams.. (total 15 pts)

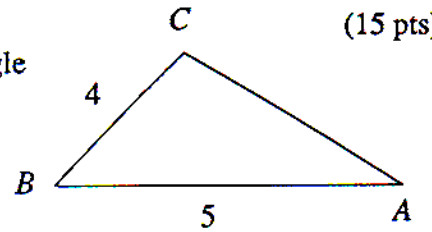
1) Derive the formula for the amount y of culture present in terms of time, t.

2) What time (to the nearest minute) will it be when the culture triples its mass?

b) The rate of decay of radium is proportional to the amount present at any time. Write a formula which describes the amount of radium which does **not** involve the constants e or k. If sixty mg of radium are present now and its half-life is 1690 years, how much radium will be present one hundred years from now? (total 15 pts)

c) Derive Newton's method for finding approximations for roots of equations. Use your derivation to find the fourth approximation x_4 if you are given $f(x) = x^4 + x - 1$ and $x_1 = .5$. (20 pts)

d) Given triangle ABC as shown on the right. The angle B is increasing at .06 radians/second. How fast is the area of the triangle changing when angle B is 60 degrees? Is the area increasing or decreasing. Will it continue to increase (or decrease) or when will it change? (15 pts)



1/12/2001

100	2
90	5
80	8
70	8
60	B
59	1

102

219/27 81.1

Very easy exam. Few enough problems that they had time to figure out them. changes...

1/24/98

Katie green 42

I $f(t) = Ae^{kt}$

$\frac{1}{2}A = Ae^{k(5730)}$

a) $e^k = \left(\frac{1}{2}\right)^{\frac{1}{5730}}$

b) $k = \frac{-\ln 2}{5730}$

c) $\frac{1}{6}A = Ae^{kt}$

$\frac{1}{6} = \left(\frac{1}{2}\right)^{\frac{t}{5730}}$

$-\ln 6 = -\frac{t}{5730}\ln 2$

$5730\frac{\ln 6}{\ln 2} = t$

$t \approx 14{,}812$ years.

The organism died 14,812 years ago.

II $y = \frac{7x+8}{x+4}$

a) $\left.\frac{dy}{dt}\right|_{x=6} = ?$ $\frac{dy}{dt} = \frac{(x+4)7\frac{dx}{dt} - (7x+8)\frac{dx}{dt}}{(x+4)^2}$

$\frac{dx}{dt} = .75$ $= \frac{(7x+28-7x-8)\frac{dx}{dt}}{(x+4)^2}$

$= \frac{20\frac{dx}{dt}}{(x+4)^2}$

$= \frac{20\cdot\frac{3}{4}}{(6+4)^2}$

$= .15$

The smog is increasing at 15% per year.

b) $\left.\frac{dy}{dt}\right|_{t=5} = ?$ $x = .08t^2 + 2$

$\frac{dx}{dt} = .16t\Big|_{t=5} = .8$

$x|_{t=5} = .08(5)^2 + 2$

$2 + 2 = 4$ in 4,000,000 people.

from part a $\frac{dy}{dt} = \frac{20}{(x+4)}\frac{dx}{dt}$

$= \frac{20}{(4+4)^2}\cdot\frac{8}{10}$

$\frac{2}{8\cdot 8}\cdot 8 = \frac{1}{4}$

so in 5 years the smog will be increasing at 25% per year.

III $G(t) = b - ce^{-kt}$

① $G(0) = b - c = 45$

② $G(5) = b - ce^{-5k} = 85$

$\lim_{t\to\infty} G(t) = 95 \therefore b = 95$ so $c = 50$ from ①

So $G(t) = 95 - 50e^{-kt}$

$G(5) = 95 - 50e^{-5k} = 85$

$10 = 50e^{-5k}$

$\frac{1}{5} = e^{-5k}$

$\left(\frac{1}{5}\right)^{\frac{1}{5}} = e^{-k}$

So $G(t) = 95 - 50\left(\frac{1}{5}\right)^{\frac{t}{5}}$ $\rightarrow G(10) = 95 - 50\left(\frac{1}{5}\right)^{\frac{10}{5}} = 95 - \frac{50}{25} = 93$

-15 minus points

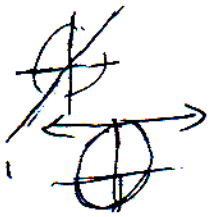
IV Polar Picture is identical to cartesian picture

Pole = origin

$(r,\theta) \to (x,y)$ $\quad x = r\cos\theta$ $\quad y = r\sin\theta$

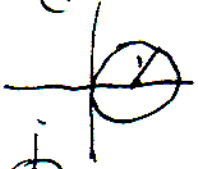
$x^2+y^2 = r^2$ $\quad \tan\theta = \frac{y}{x}$

(1) St. Line $\theta = \frac{\pi}{4}$ $\quad y = x$



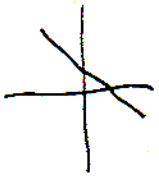
(2) St line not through pole $\quad r = \frac{1}{\sin\theta}$ $\quad y = 1$

(3) Circle centre not at O $\quad r = 2\cos\theta$ $\quad (x-1)^2+y^2 = 1$



(4) Circle centred Pole $\quad r = 1$ $\quad x^2+y^2 = 1$

(5) St line Neg Slope $\quad y = -x+1$ $\quad r = \frac{1}{\sin\theta+\cos\theta}$



(6) Circle in 2nd Quad. $\quad (x+1)^2+(y-1)^2 = 1$ $\quad r^2+2r\cos\theta-2\sin\theta = -1$

Eccentricity is the ratio of the distance from the focus to a point over the distance from that point to the directrix.

$r = \frac{1}{1+\frac{1}{2}\cos\theta}$ ellipse $e = \frac{1}{2} < 1$

$r = \frac{1}{1+\frac{3}{2}\cos\theta}$ hyperbola $e = \frac{3}{2} > 1$

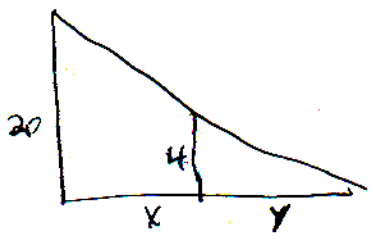
$r = \frac{1}{1+\cos\theta}$ parabola $e = 1$

$r = \frac{1}{1+0\cdot\cos\theta} = 1$ circle $e = 0$

$\frac{dy}{dt} = ?$

$\frac{dy}{dt} = -4$ (x is getting shorter)

Extra.



$\frac{x+y}{20} = \frac{y}{4}$

$x+y = 5y$

$4y = x$

$4\frac{dy}{dt} = \frac{dx}{dt}$

$\frac{dy}{dt} = \frac{1}{4}\cdot(-4)$

$= -1$

The shadow is getting shorter at 1 foot per min. No matter where he is!!

100	0
90	3
80	7
70	3
60	2
59	8

02

68.2

The 12 Days of Christmas work was needed before the eccentricity stuff. Just ended up calculating the #7 – 29 on Day of Exam. Students were so chatty... Maybe the weakest will exit....

100	2
90	3
80	4
70	3
60	2
59	7

97

$\frac{1490}{21} = 70.95$

Most did NOT relate to the exponential stuff during Paid the big price.

Great exam! Made cosmetic changes on appeals

Advanced Placement Calculus

Copy original problem.

Convince *me* that **you** understand the concept!

Name Calc Ch 11

Per _____ Date 4/11/02

97

Chapter 11 Exam

I After an organism dies, the amount of carbon-14 in the remains of the organism decreases exponentially with a half-life of 5730 years. (25 pts total)

a) Determine an expression for the constant e^k. Do not use a calculator.

b) Determine an expression for the constant k. Do not use a calculator.

c) A sample of an organism's remains contains one-sixth the amount of carbon-14 it contained when the organism died. When did it die? Write an expression for the number of years ago and then evaluate your expression on your calculator.

II Let y represent an index number that measures the smog level in a certain city. Environmentalists believe that y is a function of the city's population. In fact, they project that

$$y = \frac{7x + 8}{x + 4}$$

where x is the number of people, in millions, that will reside in the city t years from now. (25 pts tot)

a) How fast will the smog level be changing when the population is 6,000,000 if, *at that time* the population increases at the rate of 0.15 of a million per year?

b) How fast will the smog level be changing five years from now, if the population at any time t is given in millions as $x = 0.08t^2 + 2$?

III Without studying, Jake always makes a 45 on Mr. Mumaugh's tests. Whenever he studies 5 hours, he manages to make an 85. Suppose his grade on any of Mr. Mumaugh's tests is given by a function of the type $G(t) = b - ce^{-kt}$ where t represents the number of hours he studies. Also, suppose his grade approaches 95 as he increases his studying time. What will his grade be on a test for which he studies 10 hours? (25 points)

IV Describe the similarities and differences between the Cartesian and the polar graphing planes. Include in your discussion directions for converting from one form to the other and back again. List, identify and label with the cartesian as well as the polar equations: one for a straight line through the pole with positive slope, one for a straight line through the pole with negative slope, one for a straight line with positive slope NOT through the pole; one for a straight line with negative slope NOT through the pole; one for a circle with center at the pole; one for a circle with center NOT at the pole; one for a circle with center located in what would be called the "second quadrant" if we were using cartesian rather than polar labeling. Briefly describe eccentricity including examples of the equations in polar form of an ellipse, a circle, a hyperbola, and a parabola.. (tot 25 pts)

Extra Credit ---------- 5 pts ----------

Jake walks at the rate of four feet per second toward a street light whose lamp is twenty feet above the street. Jake is four feet tall. How fast is the length of this shadow changing at the moment he is

a) 12 feet from the base of the lamp post?

b) 4 feet from the base of the lamp post?

I $2y^3 + 6x^2y - 12x^2 + 6y = 1$

a) $6y^2\frac{dy}{dx} + 12xy + 6x^2\frac{dy}{dx} - 24x + 6\frac{dy}{dx} = 0$

$6(x^2+y^2+1)\frac{dy}{dx} = 24x - 12xy$

$\frac{dy}{dx} = \frac{12x(2-y)}{6(x^2+y^2+1)} = \frac{4x-2xy}{x^2+y^2+1}$

b) horizontal tangent when $\frac{dy}{dx} = 0$

$4x - 2xy \overset{\text{Set}}{=} 0$

$2x(2-y) = 0$

$x = 0 \quad y = 2$

when $x = 0$

$2y^3 + 6y = 1$

$y = .165$ using solver.

when $y = 2$

$16 + 12x^2 - 12x^2 + 12 = 1$

$28 = 1$ Reject.

there is no point on the curve when $y = 2$.

only line is $y = .165$

c) $y = -x$ tangent at P.

$\frac{dy}{dx} = \frac{4x-2xy}{x^2+y^2+1} \overset{\text{Set}}{=} -1$

$4x - 2xy = -x^2 - y^2 - 1$ $\quad y = -x$

$4x + 2x^2 = x^2 - x^2 - 1$

$= -2x^2 - 1$

$4x^2 + 4x + 1 = 0$

$(2x+1)(2x+1) = 0$

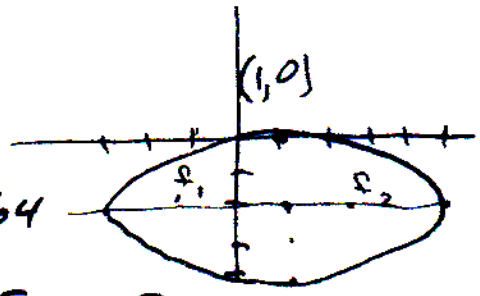
$x = -\frac{1}{2} \quad \therefore y = \frac{1}{2}$

II $4x^2 + 16y^2 = 8x - 4 - 64y$

$4(x^2 - 2x + 1) + 16(y^2 + 4y + 4) = -4 + 4 + 64$

$\frac{(x-1)^2}{16} + \frac{(y+2)^2}{4} = 1$

ellipse centr $(1, -2)$



foci: $\sqrt{16-4} = \sqrt{12} = 2\sqrt{3} = c$

So $(1-2\sqrt{3}, -2)$ $(1+2\sqrt{3}, -2)$

x-intercept $y = 0$ $(1, 0)$

y-intercept $x = 0$ $\frac{1}{16} + \frac{(y+2)^2}{4} = 1$

$\frac{(y+2)^2}{4} = \frac{15}{16}$

$(y+2)^2 = \frac{15}{4}$

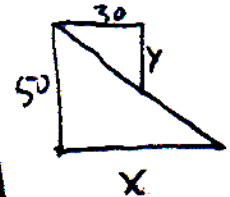
$y + 2 = \pm\frac{\sqrt{15}}{2}$

$y = -2 \pm \frac{\sqrt{15}}{2}$

$\left(0, -2 + \frac{\sqrt{15}}{2}\right)$

$\left(0, -2 - \frac{\sqrt{15}}{2}\right)$

III a) $y = 16t^2 \quad \frac{dy}{dt} = 32t$



$\frac{dx}{dt} = ?$

$t = \frac{1}{2}$

$\frac{x}{50} = \frac{30}{y}$

$\frac{1}{50}\frac{dx}{dt} = -\frac{30}{y^2}\frac{dy}{dt}$

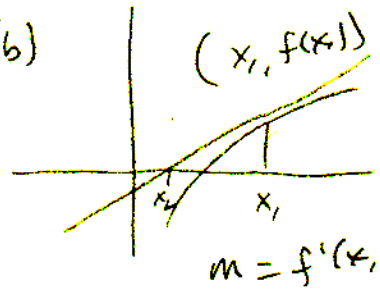
$y\Big|_{t=\frac{1}{2}} = 4 \Rightarrow y^2 = 16$

$\frac{1}{50}\frac{dx}{dt} = -\frac{30}{16} \cdot 32 \cdot \frac{1}{2}$

$\frac{dx}{dt} = -1500$

The shadow is shortening at 1,500 FT/sec.

III b)



$m = f'(x_1)$

$y - f(x_1) = f'(x_1)(x - x_1)$; x_2

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$x_{N+1} = x_N - \frac{f(x_N)}{f'(x_N)}$

$f(x) = x^4 + x - 1$

$f'(x) = 4x^3 + 1$

$x_{N+1} = x_N + \frac{x_N^4 + x_N - 1}{4x_N^3 + 1}$

$x_1 = .5$ #1

$Y_3(.5) = x_2 = .792 \Rightarrow A$ #2

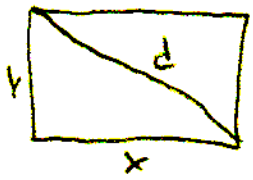
$Y_3(A) = x_3 = .730$ #3

$Y_1 = f(x)$

$Y_2 = f'(x)$

$Y_3 = x - (Y_1(x) / Y_2(x))$ $Y_3(A) = x_4 = .725$ #4

c)



$\frac{dx}{dt} = 7$

$\frac{dy}{dt} = -3$

① $\frac{dA}{dt} = ?$ $A = xy$

$\frac{dA}{dt} = y\frac{dx}{dt} + x\frac{dy}{dt}$

$= 5 \cdot 7 + 12(-3)$

$= 35 - 36$

$= -1$

getting smaller 1 unit/sec

② $\frac{dP}{dt} = ?$ $P = 2x + 2y$

$\frac{dP}{dt} = 2\frac{dx}{dt} + 2\frac{dy}{dt}$

$= 2 \cdot 7 + 2(-3)$

$= 14 - 6$

$= 8$

getting larger 8 units/sec

③ $d^2 = x^2 + y^2$ $\frac{dd}{dt} = ?$

$2d\frac{dd}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$

$13\frac{dd}{dt} = 12(7) + 5(-3)$

$\frac{dd}{dt} = \frac{84 - 15}{13} = \frac{69}{13}$ getting larger

IV Polar Plot is identical to cartesian Plot Pole = origin $(\rho, \theta) \to (x, y)$

$x = \rho\cos\theta$

$y = \rho\sin\theta$

$x^2 + y^2 = \rho^2$

$\frac{y}{x} = \tan\theta$

① ST line Thru Pole $m > 0$ $y = x$ $\theta = \frac{\pi}{4}$

② ST line Thru Pole $m < 0$ $y = -x$ $\theta = \frac{3\pi}{4}$

③ ST line $m > 0$ not Thru Pole $y = x + 1$

$r = \frac{1}{\sin\theta - \cos\theta}$

④ ST line $m < 0$ not Thru Pole $y = -x + 1$

⑤ circle centered at Pole $r = 1$ $x^2 + y^2 = 1$

⑥ circle centered at Pole $r = 2\cos\theta$ $(x-1)^2 + y^2 = 1$

⑦ circle $(x+1)^2 + (y-1)^2 = 1$

2nd Quad $r^2 + 2r\cos\theta - 2r\sin\theta = -1$

Eccentricity is the ratio of the distance from a focus to a point over the distance from that point to the directrix

$\rho = \frac{1}{1 + \frac{1}{2}\cos\theta}$ $e = \frac{1}{2}$ ∴ ellipse

$\rho = \frac{1}{1 + \frac{3}{2}\cos\theta}$ $e = \frac{3}{2}$ ∴ hyperbola

$\rho = \frac{1}{1 + \cos\theta}$ $e = 1$ ∴ Parabola

$\rho = \frac{1}{1 + 0\cdot\cos\theta} = 1$ $e = 0$ ∴ circle

1/19/02

Darryl Gaskin 48
Walter Crouch 34
Dustin Swan 30

1/18/02

Kent Boes 78
Justin Gillmas 68
Jimmy Biggs 65
Kristin Suchkonen 65
Ashley Russell 62
Cody Ballard 52
Max Espelin 47
Long Tom 36
Eastin Eqbal 35

Advanced Placement Calculus

Name Calc ch 11

Per _____ Date 11

Copy original problem.

Convince *me* that **you** understand the concept!

If decimals are used, they must be accurate to 3 places beyond the decimal.

Chapter 11 Exam

Make up Exam

01, 02

I Consider the curve defined by $2y^3 + 6x^2y - 12x^2 + 6y = 1$ (15 pts tot)

a) Show that $\frac{dy}{dx} = \frac{4x - 2xy}{x^2 + y^2 + 1}$

b) Write an equation of each horizontal tangent line to the curve.

c) The line through the orgin with slope – 1 is tangent to the curve at point P. Find the x- and y-coordinates of point P.

II Re write the given in Cartesian graphing form. Sketch the graph. Determine the coordinates of any focus (or foci) and any axis intercepts. $4x^2 + 16y^2 = 8x - 4 - 64y$ (15 pts tot)

III Word problems: (20 pts ea)

a) A light is at the top of a pole 50 feet high. A ball is dropped from the same height from a point 30 feet away from the light. Assume the ball falls $16t^2$ feet in t seconds. How fast is the shadow of the ball moving along the ground one-half second after the ball is dropped?

b) Derive Newton's method for finding approximations for roots of equations. Use your derivation to find the fourth approximation x_4 if you are given $f(x) = x^4 + x - 1$ and $x_1 = .5$.

c) The length of a rectangle is increasing at a rate of seven feet per second and the width is decreasing at a rate of three feet per second. When the length is twelve feet and the width is five feet, find the rate of change of:

1) the area 2) the perimeter 3) the diagonal

IV Describe the similarities and differences between the Cartesian and the polar graphing planes. Include in your discussion directions for converting from one form to the other and back again. List, identify and label with the cartesian as well as the polar equations: one for a straight line through the pole with positive slope, one for a straight line through the pole with negative slope, one for a straight line with positive slope NOT through the pole; one for a straight line with negative slope NOT through the pole; one for a circle with center at the pole; one for a circle with center NOT at the pole; one for a circle with center located in what would be called the "second quadrant" if we were using cartesian rather than polar labeling. Briefly describe eccentricity including examples of the equations in polar form of an ellipse, a circle, a hyperbola, and a parabola. (10 pts)

1998

another is a long list of failure by this inept group.

Calculus is easy if you have any math background. These have almost none

100	0
98	2
80	4
70	2
60	7
59	1

98

1165

16

72.8

disgusting!

1994

100	0
90	0
80	3
70	10
60	6
59	2

88

45

1482

21

70.6

Nobody got to this response as slow for problems II and

I $x = 4y^2$ $\frac{dx}{dt} = 18000$

$\frac{dy}{dt}\Big|_{y=250} = ?$

$\frac{dx}{dt} = 8y\frac{dy}{dt}$

$250000 = 4y^2$

$500 = 2y$

$y = 250$

$18000 = 8(250)\frac{dy}{dt}$

$\frac{18000}{8 \cdot 250} = \frac{dy}{dt}$

$\frac{9}{\cancel{8}} = \frac{dy}{dt}$

The company should add 9 workers per year.

II $y = Ae^{kt}$

$A = 1000$ $y = 1000(32)^t$

$t = \frac{1}{5}\ln$ $1{,}000{,}000 = 1000(32)^t$

$2A = Ae^{kt}$

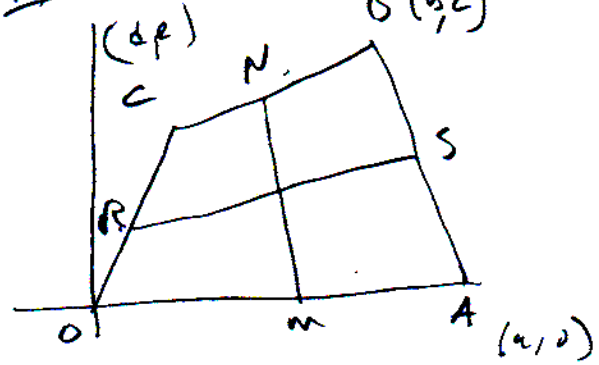
$2 = e^{k\frac{1}{5}}$ $1000 = 32^t$

$32 = e^k$ $\ln 1000 = t\ln 32$

$t = 1.99$

There will be 100000 bacteria is about 2 hrs.

III



N: $\left(\frac{b+d}{2}, \frac{c+e}{2}\right)$ S: $\left(\frac{a+b}{2}, \frac{c}{2}\right)$

R: $\left(\frac{d}{2}, \frac{e}{2}\right)$

m: $\left(\frac{a}{2}, 0\right)$

mid: $\left(\frac{a+b+d}{4}, \frac{c+e}{4}\right)$ mid $\left(\frac{a+b+d}{4}, \frac{c+e}{4}\right)$

They have the same midpoint coordinate

IV

a) $\sqrt{x} + \sqrt{y} = \sqrt{c}$

$\frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}y^{-\frac{1}{2}}\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\left(\frac{y}{x}\right)^{\frac{1}{2}}$

b) (x_1, y_1)

$m = -\left(\frac{y_1}{x_1}\right)^{\frac{1}{2}}$

$y - y_1 = -\left(\frac{y_1}{x_1}\right)^{\frac{1}{2}}(x - x_1)$

$y = -\sqrt{\frac{y_1}{x_1}}x + \sqrt{x_1 y_1} + y_1$

$x = 0$ y intercept $\sqrt{x_1 y_1} + y_1$

$y = 0$ x intercept

$\left(\frac{\sqrt{y_1}}{\sqrt{x_1}}\right)x = \sqrt{x_1 y_1} + y_1$ $\sqrt{\frac{x_1}{y_1}}$

$x = x_1 + \sqrt{x_1 y_1}$

No surface area. -4

V Polar Dict identical to cartesian P.d.
Pole = origin positive x axis = polar axis

(ρ, θ) (x, y) $x = \rho\cos\theta$ $x^2+y^2 = \rho^2$
$y = \rho\sin\theta$ $\tan\theta = \frac{y}{x}$

(1) ST lin $\theta = \frac{\pi}{4}$ $y = x$

(2) ST lin not at pole $\rho = \frac{1}{\sin\theta}$ $y = 1$

(3) circle at Pole $\rho = 1$

(4) circle with Not at pole $\rho = 2\cos\theta$ $(x-1)^2 + y^2 = 1$

Eccentricity is the ratio of the distance from a focus to a point divided by the dist from that point to the directrix

$\rho = \frac{1}{1+\frac{1}{2}\cos\theta}$ ellipse $e = \frac{1}{2} < 1$

$\rho = \frac{1}{1+\frac{3}{2}\cos\theta}$ hyperbola $e > \frac{3}{2} > 1$

$\rho = \frac{1}{1+\cos\theta}$ Parabola $e = 1$

$\rho = K$ third case $e = 0$ is no eccentricity.

Extra.

$\sqrt{x_1 y_1} + y_1 + x_1 + \sqrt{x_1 y_1} = a$. ↕ any constant.

$\therefore x_1 + 2\sqrt{x_1 y_1} + y_1 = a$

$(\sqrt{x_1} + \sqrt{y_1})^2 = a$

$\therefore (\sqrt{k})^2 = a$ Given

$k = a$

4/17/03

	4	5	T
100	0	1	
90	4	3	
80	8	4	
70	1	4	
60	1	1	
59	0	5	

102

40

1147/14 = 81 3/4

1299/18

2568/33

1269/15 = 84.6

72.2

77.8

Excellent exam.
No changes.

Advanced Placement Calculus Name Calc Ch 11

Copy original problem. Per _____ Date 2003

Convince *me* that **you** understand the concept!

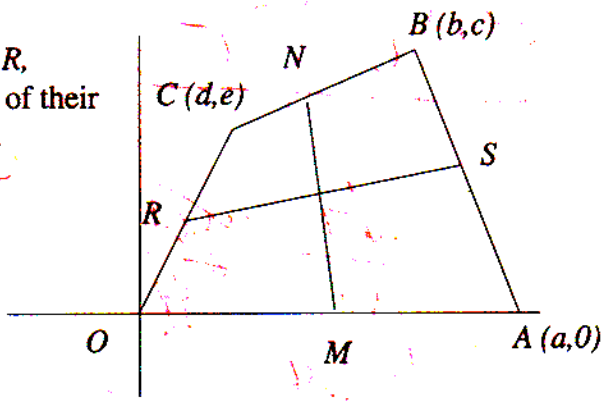
Each section 20 pts.

Chapters 11 etc. Exam

I In the manufacture of sweet bippies it is a well known fact that it takes y workers to produce x of the highly prized *sweet bippies*; in fact, the relationship is: $x = 4y^2$. If the production of the bippies this year is 250,000 units and the production is increasing at the rate of 18,000 units per year, what is the current rate at which the labor force should be increased?

II In the legendary sci-fi film, the Blob, scientists were observing alien bacteria and the gist of the dialog was that they had one thousand bacteria at 3 P.M. atnd at 3:12 P.M. the bacteria count had doubled. At about what time did the scientists expect there to be 1 million bacteria?

III Consider the figure on the right. The points, *M, S, N,* and *R,* on the sides of the quadrilateral, *OABC*, are the midpoints of their respective sides. The line $\overline{RS}$ intersects with $\overline{MN}$. Your job is to prove that that intersection point happens to be the midpoint of $\overline{RS}$ **and** the midpoint of $\overline{MN}$.



IV Given: $\sqrt{x} + \sqrt{y} = \sqrt{k}$ where k is a positive constant.

a) Find $\frac{dy}{dx}$.

b) Let (x_1, y_1) be any point on the graph of the "given". Write the equation (in graphing form) of the tangent line to the "given" that passes through that point. Identify and specifically state the value of each of the axis intercepts. (The intercepts do not need to be in coordinate format.)

V Describe the similarities and differences between the Cartesian and the polar graphing planes. Include in your discussion directions for converting from one form to the other and back again. List and identify four polar equations: one for a straight line through the pole, one for a straight line NOT through the pole; one for a circle with center at the pole; one for a circle with center NOT at the pole. List the equivalent Cartesian equation for each polar equation. Briefly describe eccentricity including examples of the equations in polar form of an ellipse, a circle, a hyperbola, and a parabola.

Extra Credit ---------- 5 pts ----------

Referring to section IV above.....

Show that the sum of the intercepts is constant and that particular constant is k.

I $w = xy + yz + xz$

4) $\frac{dw}{dt} = y\frac{dx}{dt} + x\frac{dy}{dt} + z\frac{dy}{dt} + y\frac{dz}{dt} + z\frac{dx}{dt} + x\frac{dz}{dt}$

(3) $= (y+z)\frac{dx}{dt} + (x+z)\frac{dy}{dt} + (y+x)\frac{dz}{dt}$

$x = t \quad y = t\sin t$

$\frac{dx}{dt} = 1 \quad \frac{dy}{dt} = \sin t + t\cos t$

$t = \pi$

$\frac{dy}{dt} = -\pi \quad \frac{dx}{dt} = 1 \quad \frac{dz}{dt} = -1$

$x = \pi \quad y = 0 \quad z = -\pi$

$z = t\cos t$

$\frac{dz}{dt} = \cos t - t\sin t$

$\left.\frac{dw}{dt}\right|_{t=\pi} = -\pi + 0 + \pi(-1) = -2\pi$

II Polar Pict identical to cartesian pict. Pole = origin pos x axis is Polar axis. $(\rho, \theta) \Rightarrow (x, y)$ $x = \rho\cos\theta$ $\tan\theta = \frac{y}{x}$ $y = \rho\sin\theta$ $x^2 + y^2 = \rho^2$

① $y = x$, $\theta = \frac{\pi}{4}$

② $y = -x$, $\theta = \frac{3\pi}{4}$

③ $y = x + 1$, $\rho\sin\theta = \rho\cos\theta + 1$, $\rho = \frac{1}{\sin\theta - \cos\theta}$

④ $y = -x + 1$, $\rho = \frac{-1}{\sin\theta + \cos\theta}$

⑤ $x^2 + y^2 = 1$, $\rho = 1$

⑥ $\rho = 1 + \cos\theta$, $(x-1)^2 + y^2 = 1$

⑦ $(x+1)^2 + (y+1)^2 = 1$

$x^2 + 2x + 1 + y^2 + 2y + 1 = 1$

$x^2 + y^2 + 2x + 2y = -1$

$\rho^2 + 2\rho\cos\theta + 2\rho\sin\theta = -1$

Eccentricity is The ratio of The distance from a focus over the distance from That pt to The directrix

$\rho = \frac{eq}{1 \pm e\begin{matrix}\cos \\ \sin\end{matrix}(\theta)}$

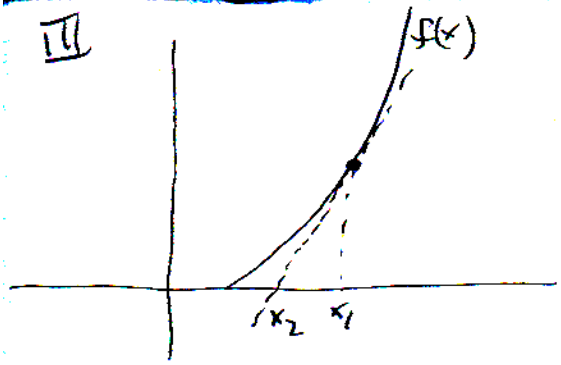
$\rho = \frac{1}{1 + \frac{1}{2}\cos\theta}$ ellipse $e = \frac{1}{2} < 1$

$\rho = \frac{1}{1 + \frac{3}{2}\cos\theta}$ hyperbola $e = \frac{3}{2} > 1$

$\rho = \frac{1}{1 + \cos\theta}$ Parabola $e = 1$

$\rho = \frac{1}{1 + 0\cos\theta}$ circle $e = 0$

IV



x_1 is 1st guess x_2 is x intercept of tangent line

$y - f(x_1) = f'(x_1)(x - x_1)$ where $y = 0$ $x = x_2$

$-f(x_1) = f'(x_1)(x_2 - x_1)$ one guess leads to the next & so on

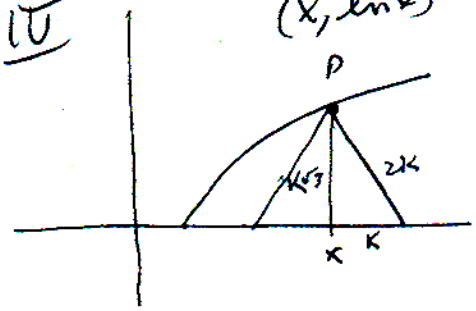
$\frac{-f(x_1)}{f'(x_1)} = x_2 - x_1$ You are using the tangent line to approx the curve and, to yield better guesses

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

if $f' = 0$ formula doesn't work because f is a constant and doesn't cross x axis or is the x axis.

When $f'' = 0$ and $f' \neq 0$ the tangent line is the f'n and exact crossing occurs on first guess.

IV



$\frac{dx}{dt} = \sqrt{3}$

$\frac{dA}{dt} = ?$

$A = \frac{1}{2}bh$

$= \frac{1}{2} 2K \ln x$

$= \frac{\ln x}{\sqrt{3}} \cdot \ln x = \frac{(\ln x)^2}{\sqrt{3}}$

$\ln x = K\sqrt{3}$

$K = \frac{\ln x}{\sqrt{3}}$

$(x, \ln x)$

$\ln x = 1$

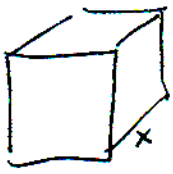
$x = e$

$\frac{dA}{dt} = \frac{2}{\sqrt{3}} \frac{\ln x}{x} \frac{dx}{dt}\Big|_{x=e}$

$\frac{2}{\sqrt{3}} \frac{1}{e} \cdot \sqrt{3} = \frac{2}{e}$

The area is increasing at $\frac{2}{e}$ sq units/sec

V



$V = x^3$

$S = 6x^2$

$\frac{dS}{dt} = \frac{-1}{2}$

$\frac{dV}{dt}\Big|_{x=3} = ?$

(A)

$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$

$= \frac{3x^2}{-24x}$

$= -\frac{x}{8}\Big|_{x=3}$

$= -\frac{3}{8}$

Volume is decreasing at the rate of $\frac{3}{8}$ of a cu inch per minute

$54 = 6x^2$

$x^2 = 9$

$x = 3$

$\frac{dS}{dt} = 12x \frac{dx}{dt}$

$\frac{-1}{24x} = \frac{dx}{dt}$

B) $\frac{df}{dt}\Big|_{x=3} = ?$

$f = x^2$

$\frac{df}{dt} = 2x \frac{dx}{dt}$

$x \frac{dx}{dt} = \frac{-1}{24}$

So $\frac{df}{dt} = -\frac{1}{12}$

area of face is decreasing at $\frac{1}{12}$ sq" per min

C) $\frac{1}{2}$ / min : 54 $\Rightarrow$ 108 min

1 hr 48 min

cube disappears at 4:48

1/27/95

Scott Bell '74

Courtney Border 65

Jennifer Spradlin 63

1/17/03

Erin Karsnes 95

Anita Hemavian 93

Jenna Fussell 90

Nicole Parsons 78

Rebecca Glover 63

Brad Banks 50

Minimum < 80

David Lund. 86

Advanced Placement Calculus

Name _____

Copy original problem. Per _____ Date _____

Convince *me* that **you** understand the concept!

95/03

Chapter 11 Make–Up Exam

I Given $w = xy + yz + xz$ where w, x, y and z are functions of t. (10 pts ea)

A. Find $\frac{dw}{dt}$

B. Given $x = t$; $y = t\sin t$; $z = t\cos t$ and your answer to part A ...

Find $\frac{dw}{dt}$ when $t = \pi$

II Describe the similarities and differences between the Cartesian and the polar graphing planes. Include in your discussion directions for converting from one form to the other and back again. List, identify and label with the cartesian as well as the polar equations: one for a straight line through the pole with positive slope, one for a straight line through the pole with negative slope, one for a straight line with positive slope NOT through the pole; one for a straight line with negative slope NOT through the pole; one for a circle with center at the pole; one for a circle with center NOT at the pole; one for a circle with center located in what would be called the "second quadrant" if we were using cartesian rather than polar labeling. Briefly describe eccentricity including examples of the equations in polar form of an ellipse, a circle, a hyperbola, and a parabola. (20 pts)

III Explain how "Newton's method" solves equations. Draw a picture, derive the recursion identity. *Include* a discussion of the significance of (1) when $f'(x) = 0$ and (2) when $f''(x) = 0$ and $f'(x) \neq 0$ (10 pts)

Related rates. Draw a picture, identify required items, solve, answer using complete sentences.

IV A point P with coordinates (x, y) moves along the curve $y = \ln x$ so that its abscissa value (x–coordinate) increases at the rate of $\sqrt{3}$ inches per second. If P is one vertex of an equilateral triangle whose other two verticies are on the x–axis, find out how fast the area of the triangle is changing when the ordinate (y–coordinate) value is one. (25 pts)

V A cube is contracting so that its surface area decreases at the constant rate of one-half square inch per minute. (25 pts total)

A. Determine how fast the volume is changing when at 3 P.M. the surface area is fifty-four square inches.

B. Determine how fast the area of one of the faces is changing at the same time.

C. Determine when will the cube disappear?

Calculus
Chapter 11, etc.
2002 – 2003

			Exam Ch 6 Applications 12/ 12	# 1 Pg 441 1 - 15 12/ 13
# 2 Pg 442 20, 26 – 28 Skip 26 "c" 12/16	# 3 Pg 448-449 1 – 7 Do problem 3C two ways. 1) use $\tan\theta = \frac{x}{3}$ 2) use $\theta = \tan^{-1}\frac{x}{3}$ 12/17	# 4 Pg 402 1 – 5, 9 – 11, 13 12/18	# 5 Pg 468 GQ Ch 11 1 – 4 Pg 469 RE 5 – 7 on pblm 5 find y' at $(\pi, 1)$ only. 12/19	Minimum Day 12/ 20
	During Christmas Vacation you are to prepare the Twelve Days of Christmas (on the back of this HW sked. Ho Ho Ho.) # 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16,17			***Vacation HW is due Jan 6. If you are unable to be in class then your work must be postmarked on or before Jan 6th.***
Turn in 12 Days of Christmas **"Electric Eccentricity"** # 18, 19, 20, 21 1/ 6	**Continue on Electric Eccentricity** 1/7	1/ 8	turn in **"Electric Eccentricity" tomorrow** 1/ 9	# 22 Pg 469 9–14 on pblm 11 find y' at (1, 2) and y' at (1, 1) **Review Ex** 1/10
# 23 Pg 469 – 470 15,16, 18, 19, 22, 24 –27, 29 **Review Ex** 1/13	# 24 Pg 396 2 – 16 even 20 – 30 even 1/14	# 25 Pg 923 1 – 7 1/15	Put last year's exam on the boards. Come to class ***PREPARED!!*** 1/16	**Chapter 11 etc. EXAM**
School Holiday 1/ 20	Exams Return. Quarter grades issued. 1/21	**Final Exam Per 1, 2** 1/22	**Final Exam Per 3, 4** 1/23	**Final Exam Per 5, 6** 1/24

I A) $w = xy + yz + xz$

$$\frac{dw}{dt} = y\frac{dx}{dt} + x\frac{dy}{dt} + z\frac{dy}{dt} + y\frac{dz}{dt} + z\frac{dx}{dt} + x\frac{dz}{dt}$$

B) $$\frac{dw}{dt} = (y+z)\frac{dx}{dt} + (x+z)\frac{dy}{dt} + (x+y)\frac{dz}{dt}$$

not enough −3

$$= (-\pi) + 0 + (-\pi) = -2\pi$$

$x = t \quad y = t\sin t$

$\frac{dx}{dt} = 1 \quad \frac{dy}{dt} = \sin t + t\cos t$

$x = \pi \quad y = 0$

$\frac{dy}{dt} = -\pi$

$z = t\cos t$

$\frac{dz}{dt} = \cos t - t\sin t$

$z = -\pi$

$\frac{dz}{dt} = -1$

II

A) $x^2 + (y-3)^2 = 9$

$\rho^2\cos^2\theta + \rho^2\sin^2\theta - 6\rho\sin\theta + 9 = 9$

$\rho^2 - 6\rho\sin\theta = 0$

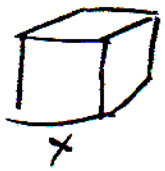
$\rho = 6\sin\theta$

B) $r = \dfrac{3}{\cos\theta + \sin\theta}$

$r\cos\theta + r\sin\theta = 3$

$x + y = 3$

III A)



S = surface area

$\frac{dS}{dt} = -\frac{1}{2} \quad \frac{dv}{dt} = ?$

$6x^2 = S$

$x = \left(\frac{S}{6}\right)^{\frac{1}{2}}$

$V = x^3$

$V = \left(\frac{S}{6}\right)^{\frac{3}{2}}$

$\frac{dv}{dt} = \frac{3}{2}\left(\frac{S}{6}\right)^{\frac{1}{2}} \cdot \frac{1}{6} \cdot \frac{dS}{dt}$

$= \frac{3}{2}(9)^{\frac{1}{2}}\frac{1}{6}\left(\frac{-1}{2}\right)$

$= \frac{3}{2} \cdot \cancel{9}^{3} \cdot \frac{1}{\cancel{6}_{2}}\left(\frac{-1}{2}\right)$

$= -\frac{3}{8}$

~~$6x^2 = 54$~~

~~$x^2 = 9$~~

~~$x = 3$~~

Not Needed.

B)

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\frac{h^2}{4} \cdot h$

$V = \frac{\pi}{12}h^3$

$\frac{h}{r} = \frac{4}{2}$

$h = 2r$

$r = \frac{h}{2}$

$\frac{dv}{dt} = \frac{\pi}{4}h^2\frac{dh}{dt}$

$\frac{dv}{dt} = -\frac{1}{2}$

$-\frac{1}{2} = \frac{\pi}{4}h^2\frac{dh}{dt}$

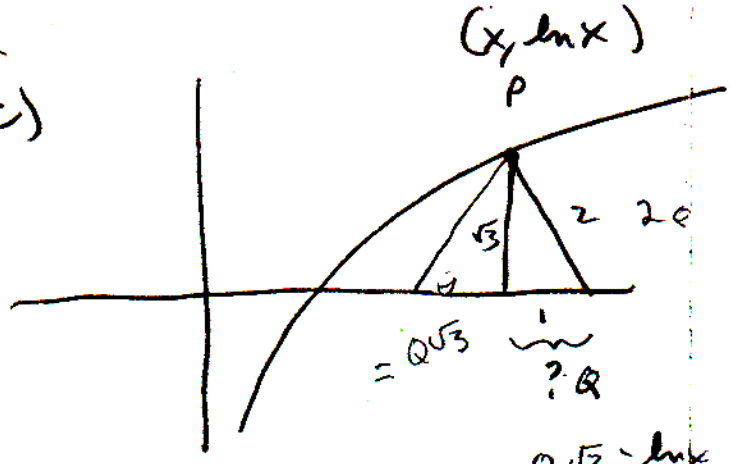
$\frac{dh}{dt} = ?$

$h = ? \quad r = 1 \; ; \; h = 2$

$-\frac{1}{2} = \pi\frac{dh}{dt}$

$\frac{dh}{dt} = -\frac{1}{2\pi}$

III c)



$\frac{dx}{dt} = \sqrt{3}$ $\quad \frac{dA}{dt} = ?$

when $y = 1$ $x = e$

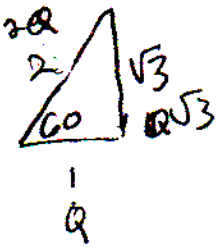
$y = \ln x$

$1 = \ln e$

$A = \frac{1}{2} b h$

$A = \frac{1}{2} x \ln x \ln x$

$A = \frac{(\ln x)^2}{\sqrt{3}}$



$Q\sqrt{3} = \ln x$

$Q = \frac{\ln x}{\sqrt{3}}$

$Q\sqrt{3} = \ln x$

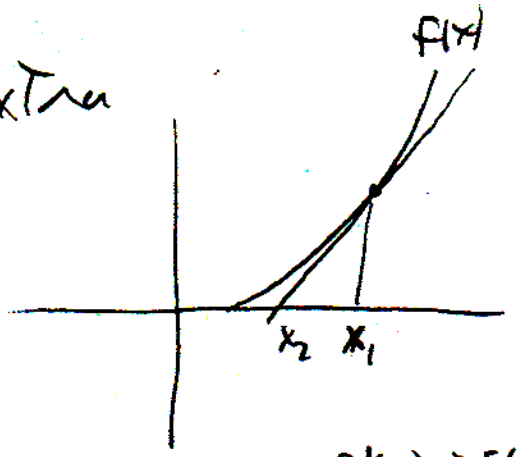
$Q = \frac{\ln x}{\sqrt{3}}$

$\frac{dA}{dt} = \frac{2 \ln x}{\sqrt{3}} \cdot \frac{1}{x} \cdot \frac{dx}{dt}$ (evaluated at $x = e$)

$= \frac{2}{\sqrt{3}} \cdot \frac{1}{e} \cdot \sqrt{3}$

$= \frac{2}{e}$

EXTRA



x_1 is 1st guess $f'(x_1)$ is slope x_2 is x intercept of tangent line.

$y - f(x_1) = f'(x_1)(x - x_1)$ when $y = 0$ $x = x_2$

$-f(x_1) = f'(x_1)(x_2 - x_1)$

$\frac{-f(x_1)}{f'(x_1)} = x_2 - x_1$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$

One guess leads to the next & so on, You are using the tangent lines to approximate the curve, and yield better "guesses"

when $f' = 0$ formula doesn't work because f is a constant and doesn't cross x axis.

when $f'' = 0$ and $f' \neq 0$, the tangent line is the fctn and exact crossing occurs on first guess.

91

100	0
90	8
80	7
70	7
60	2
54	7

99

39

$\frac{2073}{26} = 79.73$

Excellent level exam.

2 working for synthesis didn't take exam

looks like 4 more should

87

100	3
90	4
80	6
70	1
60	6
54	4

78.77

bottom 10 are in the wrong course! other mist – Sepm results.

III A needs to be worded better

also I needs change as shown.

Advanced Placement Calculus AB Name calc ch11 1991

Copy the original problem.
Convince *me* that **you** understand the concept. Per _____ Date 87

CHAPTER #11

I Given: $W = XY + YZ + XZ$ where W, X, Y and Z are functions of t. (10 pts ea)

A) Find $\frac{dW}{dt}$

B) Given: $X = t$; $Y = t \operatorname{Sin} t$; $Z = t \operatorname{Cos} t$; and your answer to part A...

Find $\frac{dW}{dt}$ when $t = \pi$.

II Polar and Cartesian (10 pts ea)

A) Determine the simpliest polar equation (ie. $\rho = \ldots.$) which is equivalent to $x^2 + (y-3)^2 = 9$

B) Determine the cartesian equlivalent to $\rho = \frac{3}{\operatorname{Cos}\theta + \operatorname{Sin}\theta}$

III Related rates. Draw a picture, identify required items and solve. (20 pts ea)

A) A cube is contracting so that its surface area **decreases** at the constant rate of one-half square inches per second. Determine how fast the volume is changing when the surface area is fifty-four square inches.

B) The diameter and height of a paper cup in the shape of a cone are both four inches. Water is leaking out at the rate of one-half cubic inches per second. Find the rate at which the water level is dropping when the **diameter** of the surface of the water is two inches.

Volume of a cone: $V = \frac{1}{3}\pi r^2 h$

C) A point P with coordinates (x, y) moves along the curve $y = \operatorname{Ln} x$ so that its x-coordinate increases at the rate of $\sqrt{3}$ inches per second. If P is one vertex of an equilateral triangle whose other two verticies are on the x-axis, find out how fast the area of the triangle is changing when the y-coordinate of P is one.

Extra Credit ——— **5 pts** ———

Explain how "Newton's method" solves equations. Draw a picture, derive the recursion identity. Include a discussion of the significance of:

1) when $F' = 0$
2) when $F'' = 0$ but $F' \neq 0$

I

$\frac{dr}{dt} = ?$ $\quad h = 4r$

$\frac{dV}{dt} = 24\pi$ $\quad \frac{dh}{dt} = 4\frac{dr}{dt}$

$r = \frac{3}{2}$ $\quad h = 4\left(\frac{3}{2}\right) = 6$

$V = (\pi r^2)h$ $\quad$ or $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 2\pi r h \frac{dr}{dt} + \pi r^2 \frac{dh}{dt}$ $\quad \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$24\pi = 2\pi r h \frac{dr}{dt} + 4\pi r^2 \frac{dr}{dt}$ $\quad 24\pi = 4\pi(3)^2 \frac{dr}{dt}$

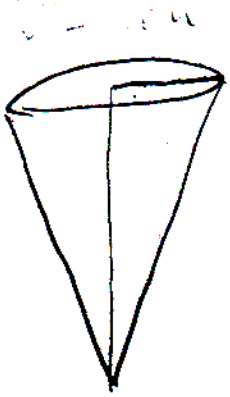
$\frac{24\pi}{2\pi r h + 4\pi r^2} = \frac{dr}{dt}$ $\quad \frac{8\pi}{9} = \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{24}{2\left(\frac{3}{2}\right)\cdot 6 + 4\left(\frac{3}{2}\right)^2}$

$\frac{24}{18+9} = \frac{24}{27} = \frac{8}{9} = \frac{dr}{dt}$

radius is increasing at $\frac{8}{9}$ in/sec.

II



$r = \frac{1}{3}h$ $\quad V = \frac{1}{3}\pi r^2 h$

$\frac{dV}{dt} = 10$ $\quad \left|\frac{dh}{dt}\right| = ?$ $\quad V = 1000$

$V = \frac{1}{3}\pi h\left(\frac{h^2}{9}\right)$ $\quad \frac{dV}{dt} = \frac{\pi h^2}{9}\frac{dh}{dt}$

$V = \frac{1}{3}\pi\frac{h^3}{9}$

$V = \frac{\pi h^3}{27}$ $\quad 90 = \pi\left(\frac{30}{\sqrt[3]{\pi}}\right)^2 \frac{dh}{dt}$

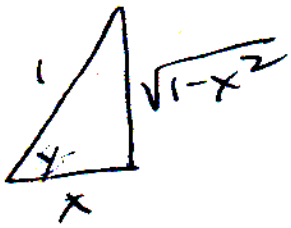
$1000 = \frac{\pi h^3}{27}$ $\quad \frac{90}{30\cdot 30}\cdot\frac{\sqrt[3]{\pi^2}}{\pi} = \frac{dh}{dt}$

$h^3 = \frac{27000}{\pi}$ $\quad \frac{1}{10\sqrt[3]{\pi}} = \frac{dh}{dt}$

$h = \frac{30}{\sqrt[3]{\pi}}$

III

$y = \cos^{-1}x$



A) $\cos y = x$

B) $-\sin y \frac{dy}{dx} = 1$

$\frac{dy}{dx} = -\frac{1}{\sin y} = -\csc y$

from picture.... $\frac{-1}{\sqrt{1-x^2}}$

IV

A) $y = \tan(\arctan(\sin x))$

$= \sin x$

$y' = \cos x$

B) $y = \sin x^{\cos x} = \sin e^{\ln x^{\cos x}}$

$\cos x \ln x$

$y = \sin e$

$y' = \cos e^{\cos x \ln x}\left[-\ln x \sin x + \frac{\cos x}{x}\right]$

$y' = \cos x^{\cos x}\left[\frac{\cos x}{x} - (\sin x)\ln x\right]$

or using ln

$y' = (\cot x \cos x - \sin x \ln \sin x)\sin x^{\cos x}$

C) $y = x^4 \ln e^{2x}$

$= 2x^5$

$y' = 10x^4$

D) $y = e^{\cos^2 x}$

$y' = e^{\cos^2 x}(2\cos x(-\sin x))$

$y' = -2e^{\cos^2 x}\sin x\cos x$

Polar pict identical to Cartesian pict
Pole = origin positive x axis = polar axis

(ρ, θ) (x, y) $x = \rho\cos\theta$ $y = \rho\sin\theta$ $x^2+y^2=\rho^2$ $\tan\theta = \frac{y}{x}$

① st lines $\theta = \frac{\pi}{4}$; $y = x$

② st line not through pole $\rho = \frac{1}{\sin x}$ $y = 1$

③ circle centre not at pole $\rho = 2\cos\theta$ $(x-1)^2+y^2=1$

④ circle $\rho = 1$ $x^2+y^2=1$

Eccentricity = The ratio of the distance from the focus to a point on the dist from that point to the directrix

$\rho = \frac{1}{1+\frac{1}{2}\cos x}$ ellipse $e = \frac{1}{2} < 1$

$\rho = \frac{1}{1+\frac{3}{2}\cos x}$ hyperbola $e = \frac{3}{2} > 1$

$\rho = \frac{1}{1+\cos x}$ parabola $e = 1$

$\rho = \frac{1}{1+0\cos x} = 1$ circle $e = 0$

Extra Volume $\pi r^2 h = V$ $1\frac{1}{2}" = \frac{1}{8}$ foot $r = 1$

$V = \pi h$

$V + \Delta V = \pi(h+\frac{1}{8})$

$= \underbrace{\pi h} + \frac{\pi}{8}$

$V + \Delta V = V + \frac{\pi}{8}$

∴ Vol of Stone is $\frac{\pi}{8}$

100 – 2
90 – 0
80 – 6
70 4
60 2
59 1

100 (2 of them)

54

74.9

a very easy exam. – and awful results.
the top 6 are probably keepers and
the bottoms are here to keep the class
count high enough to keep the course
offering. Yuk.

Advanced Placement Calculus AB Name Calc C 11 1992

Copy the original problem.
Convince *me* that **you** understand the concept. Per _____ Date _____

CHAPTER #11

I A metallic right circular cylinder is continuously heated. If the height is four times the radius, what is the rate of change of the radius when the volume is increasing at the rate of 24π in/sec and the radius is one and one-half inches? ($V_{cylinder} = \pi r^2 h$) (20 pts tot)

II A conical reservoir with radius equal to one-third of its height, is filling with water at the rate of ten cubic feet per second. Show that the water level is rising at a rate of $\frac{1}{10}(\pi)^{-\frac{1}{3}}$ when there is one thousand cubic feet of water in the tank? (hint: what is 'h' when 'v' is one thousand?) (20 pts)

$$V_{cone} = \tfrac{1}{3}\pi r^2 h$$

III Given: $y = \cos^{-1} x$ (20 pts tot)

A) Solve for x. B) Using implicit differentiation, show that $\frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}}$

IV Given y, find $\frac{dy}{dx}$. (Final answer must *not* include the letter y.) (5 pts ea)

A) $y = \tan(\arctan(\sin x))$ B) $y = \sin x^{\cos x}$ C) $y = x^4 \ln e^{2x}$ D) $y = e^{\cos^2 x}$

V Describe the similarities and differences between the Cartesian and the polar graphing planes. Include in your discussion directions for converting from one form to the other and back again. List and identify four polar equations in the form of r = ...; one for a straight line through the pole, one for a straight line NOT through the pole; one for a circle with center at the pole; one for a circle with center NOT at the pole. List the equivalent Cartesian equation for each polar equation. Briefly describe eccentricity including examples of the equations in polar form of an ellipse, a circle, a hyperbola, and a parabola. (tot 20 pts)

Extra Credit — 5 pts

A stone is dropped into a cylindrical water tank which is two feet in diameter, causing the water level to rise one and one-half inches. What is the volume of the stone?

I

$\frac{dv}{dt} = -15$ $\quad V = \frac{4}{3}\pi r^3$

$\frac{dx}{dt} = ?$ $\quad V = \frac{4}{3}\pi (10+x)^3$

$\frac{dv}{dt} = 4\pi (10+x)^2 \frac{dx}{dt}$

The Thickness is decreasing at $\frac{1}{60\pi}$ " per minute.

$\frac{-15}{4\pi(10+x)^2} = \frac{dx}{dt}$ $\quad x=5$

$\frac{-15}{4\pi \cdot 15^2} = \frac{dx}{dt}$

$\frac{dx}{dt} = \frac{-1}{60\pi} = -.0053$

II

$\frac{d\theta}{dt} = 6\pi$ $\quad \tan\theta = \frac{8}{\omega_0}$

$\frac{d\theta}{dt} = ?$ $\quad \sec^2\theta \frac{d\theta}{dt} = \frac{1}{\omega_0}\frac{dg}{dt}$

$\theta = \frac{\pi}{4}$ $\quad 2 \cdot 1^2 \pi = \frac{1}{\omega_0}\frac{dg}{dt}$

$\frac{dg}{dt} = 2000\pi = 6283.1853$

The spot of light is moving at 2000π Rad/min

III

$(y+1)^2 = 3x^2+1$

$2(y+1)\frac{dy}{dx} = 6x$

$\frac{dy}{dx} = \frac{3x}{y+1}$

$(y+1)^2 = 3(1)^2+1$

$(y+1)^2 = 4$

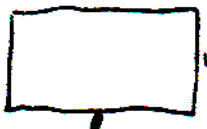
$y+1 = \pm 2$

$y = \{-3, 1\}$ $\quad x=1$

$\frac{dy}{dx} = \frac{3}{-2}$ or $\frac{3}{2}$

$\frac{dy}{dx} = \left\{\frac{-3}{2}, \frac{3}{2}\right\}$

IV



$\frac{dl}{dt} = 7$ $\quad \frac{dA}{dt} = ?$

$\frac{dw}{dt} = -3$ $\quad A = wl$

$\frac{dA}{dt} = l\frac{dw}{dt} + w\frac{dl}{dt}$

The area is decreasing at 1 sq ft Per Second.

$= 12(-3) + 5(7)$

$= -36 + 35$

$= -1$

b) $\frac{dP}{dt} = ?$

$P = 2l + 2w$

$\frac{dP}{dt} = 2\frac{dl}{dt} + 2\frac{dw}{dt}$

$= 2 \cdot 7 + 2(-3)$

$= 14 - 6$

$= 8$

Perimeter is growing at 8 ft Per Second.

c) $\frac{dA}{dt} = ?$

$d^2 = w^2 + l^2$

$2d\frac{dd}{dt} = 2w\frac{dw}{dt} + 2l\frac{dl}{dt}$

$\frac{dd}{dt} = \frac{w\cdot\frac{dw}{dt} + l\frac{dl}{dt}}{d}$

$= \frac{5(-3)+12(7)}{13}$

$= \frac{-15+84}{13}$

$d = 12^2 + 5^2$

$d = 13$

$\frac{dd}{dt} = \frac{69}{13}$

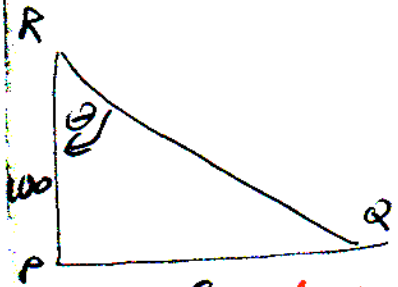
dial getting longer at $\frac{69}{13}$ in Per Sec.

$\frac{dS}{dt} = ?$ $\quad S = 4\pi r^2$ $\quad \frac{dr}{dt} = \frac{dy}{dt}$

$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$ $\quad r = 10+x$

$\frac{dS}{dt} = -\frac{8\pi(10+x)^2}{60\pi}$ $\quad x=5$

$\frac{dS}{dt} = -2$ The surface area is decreasing at 2 sq " Per min.



-12 if Q took of evolution pt better STOPS at all

use evolution PT and still from right track $= -8\pi$

V Polar pict identified to cartesian pict

Pole = origin. pos x-axis = polar axis.

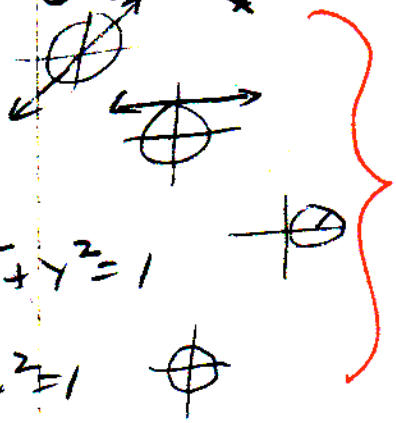
(ρ, θ) (x, y). $x = \rho\cos\theta$ $\quad y = \rho\sin\theta$ $\quad x^2+y^2 = \rho^2$ $\quad \tan\theta = \frac{y}{x}$

① st line $\theta = \frac{\pi}{4}$; $y = x$

② st line not through pole $\rho = \frac{1}{\sin\theta}$; $y = 1$

③ circle centre not at pole $\rho = 2\cos\theta$ $\quad (x-1)^2 + y^2 = 1$

④ circle centre at pole $\rho = 1$ $\quad x^2+y^2 = 1$



10

Eccentricity is the ratio of the distance from a focus to a point on the dist from that point to the directrix.

5 PTS

$\rho = \frac{1}{1+\frac{1}{2}\cos\theta}$ ellipse $e = \frac{1}{2} < 1$

$\rho = \frac{1}{1+\frac{3}{2}\cos\theta}$ hyperbola $e = \frac{3}{2} > 1$

$\rho = \frac{1}{1+\cos\theta}$ parabola $e = 1$

$\rho = \frac{1}{1+0.\cos\theta} = 1$ circle $e = 0$

Extra.

speed $= v$

gas $= \frac{1}{1600}v^2 + \frac{3}{40}v$ l/hr.

dist $= 160$

time $= RT = D$ $\quad \therefore \frac{160}{v}$ hrs.

$\tau = \frac{D}{R}$

gas used $= \frac{160}{v}\left[\frac{1}{1600}v^2 + \frac{3}{40}v\right]$ $\quad 8¢/\frac{1}{10}$ litre $\quad 80$/litre

$f(v) = \left(30 - \frac{160}{v}\left[\frac{1}{1600}v^2 + \frac{3}{40}v\right]\right)80$ — less Penalty. $\left(-\frac{160}{v} + 60 + 120,\ 3\right)$ PTS

gas left $\quad$ 8¢ per litre $\quad$ minutes $\quad$ 2 hrs $\quad$ time in excess of 2 hrs.

$f(v) = \left[30 - \frac{v}{10} + 120\right]80 - 3.60\left(\frac{160}{v} + 2\right)$

$f(v) = 18180 - 8v - 360\left(\frac{80}{v} + 1\right)$

$f(v) = 1440 - 8v + \frac{28800}{v} + 360$

$f'(v) = -8 + \frac{28800}{v^2} \overset{\text{set}}{=} 0$

$v^2 = 3600$

$v = 60$ Reject $v = -60$.

$f''(v) = -\frac{57600}{v^3} < 0$

$\therefore$ max at $v = 60$ f'' test.

93

100 0

90 8

80 4

70 5

60 3

59 Q.

$\frac{1732}{21} = 82.5$

Better Than
Expected results.
& really can't
account for it.
We did put practically
Every two on Brand up to
Elective Eccentricity.
That Still needs work.
eg. why e=0 means circle?
because there is not "off centre"
both foci coincide at
a point called the centre

1995

100	0
90	5
80	4
70	2
60	2
59	3

1167 = 16

77.8

left Klein cant sevis

Advanced Placement Calculus AB Name 1/20/95

Copy the original problem.
Convince *me* that **you** understand the concept. Per _____ Date 93

CHAPTER #11

I A spherical iron ball 20 inches in diameter is coated with a layer of ice of uniform thickness. Assuming that the ice is melting at the rate of 15 ***cubic*** inches per minute, how fast is the thickness of the ice decreasing when it is 5 inches thick? How fast is the outer surface area of the ice decreasing at this instant? *Volume of Sphere* $= \frac{4}{3}\pi r^3$ *Surface Area of Sphere* $= 4\pi r^2$ (20 pts tot)

Figure for **I:**

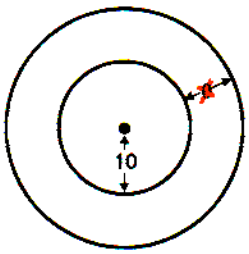
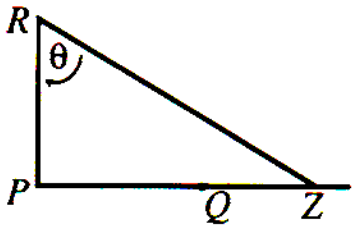


Figure for **II:**



II A spotlight is rotating so that its light shines parallel to the ground. It rotates 5 times every minute. (i.e. $\frac{d\theta}{dt} = 5 \cdot 2\pi = 10\pi$ radians/minute.) Point P is on wall PZ, 100 feet from the light R. See the picture above. PR = PQ = 100 feet. How fast is the beam of light moving along the wall at the instant that the spot of light is shining on point Q? (20 pts)

III Given: $(y+1)^2 = 3x^2 + 1$. Find: $\frac{dy}{dx}$ when $x = 1$. (Why is this a 20 point problem?) (20 pts)

IV The length of a rectangle is increasing at a rate of seven feet per second and the width is decreasing at a rate of three feet per second. When the length is twelve feet and the width is five feet, find the rate of change of: (20 pts tot)

a) the area b) the perimeter c) the diagonal

V Describe the similarities and differences between the Cartesian and the polar graphing planes. Include in your discussion directions for converting from one form to the other and back again. List and identify four polar equations: one for a straight line through the pole, one for a straight line NOT through the pole; one for a circle with center at the pole; one for a circle with center NOT at the pole. List the equivalent Cartesian equation for each polar equation. Briefly describe eccentricity including examples of the equations in polar form of an ellipse, a circle, a hyperbola, and a parabola. (tot 20 pts)

Extra Credit ═══ 5 pts ═══

You are one of the contestants in an economy car rally. At the starting line you will get 30 liters of gasoline in the tank. If you drive with velocity v km/hr, then your car consumes $\frac{1}{1600}v^2 + \frac{3}{40}v$ liters per hour. The route is 160 km long and if you take more than 2 hours, then you get penalized 3 points for each additional minute you take. However, this may not matter too much because you also get a reward for economical driving. Specifically, you get 8 points for each deciliter (one-tenth of a liter) left in the tank when you finish. How fast should you drive to get the most points?

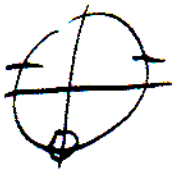
I vertical asymptote when denom = 0

$-r\sin\theta + r'\cos\theta$ $\quad r = 1+\sin\theta$, $r' = \cos\theta$

$D = -(1+\sin\theta)\sin\theta + \cos^2\theta$ set

$-\sin\theta - \sin^2\theta + \cos^2\theta = 0$

$+1 - \sin^2\theta =$



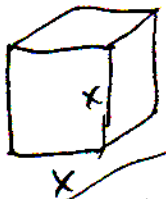
$2\sin^2\theta + \sin\theta - 1 = 0$

$(2\sin\theta - 1)(\sin\theta + 1) = 0$

$\sin\theta = \frac{1}{2}$ $\quad \sin\theta = -1$

$\theta \in \{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}\}$

II



$\frac{dv}{dt} = K(6x^2)$

WTF $\frac{dA}{dt} = ?$

$A = x^2$ area of a side

$V = x^3$ $\quad \frac{dA}{dt} = 2x\frac{dx}{dt}$

$\frac{dv}{dt} = 3x^2\frac{dx}{dt}$

$6Kx^2 = 3x^2\frac{dx}{dt}$

$2K = \frac{dx}{dt}$

Since x is edge of cube, $\frac{dx}{dt}$ = a constant

$\frac{dA}{dt} = 2x\frac{dx}{dt}$

$= 4Kx$

$\frac{dA}{dt}$ Not constant

$\frac{dx}{dt}$ is constant.

III $2x\sin y + y\cos x - 2y = 0$

$2\sin y + 2x\cos y\frac{dy}{dx} + \cos x\frac{dy}{dx} + y\sin x - 2\frac{dy}{dx} = 0$

$2(1) + 2(\frac{\pi}{2})\cdot 0\frac{dy}{dx} + 0\cdot\frac{dy}{dx} + \frac{\pi}{2}(1) - 2\frac{dy}{dx} = 0$

$2 - \frac{\pi}{2} = 2\frac{dy}{dx}$

$1 + \frac{\pi}{4} = \frac{dy}{dx}$

IV

Vase − Room = Difference $= Ae^{kt}$

$D = 50(\frac{3}{4})^t$

$\frac{3}{4}A = Ae^{k(4)}$

$D(8) = 50(\frac{3}{4})^8 \approx 5^0$

$\frac{3}{4} = e^k$

Room's constant

$\therefore$ Vase is $70+5$

75^0.

$T_{VASE}(t) = 70 + 50(\frac{3}{4})^t$

$K = \ln(\frac{3}{4})$

V Polar pict is identical

Pole = origin Pos x-axis = polar axis.

(ρ, θ) (x, y) $x = \rho\cos\theta$ $x^2+y^2=\rho^2$

$y = \rho\sin\theta$ $\tan\theta = \frac{y}{x}$

① ST line $\theta = \frac{\pi}{4}$; $y = x$

② ST line not through Pole $\rho = \frac{1}{\sin\theta}$ $y = 1$

③ circle cntr not at Pole $\rho = 2\cos\theta$ $(x-1)^2+y^2=1$

④ circle cntr at Pole $\rho = 1$ $x^2+y^2=1$

Eccentricity is the ratio of the distance from the focus to a point over the distance from that point to the directrix

$\rho = \frac{1}{1+\frac{1}{2}\cos\theta}$ ellipse $e = \frac{1}{2} < 1$

$\rho = \frac{1}{1+\frac{3}{2}\cos\theta}$ hyperbola $e = \frac{3}{2} > 1$

$\rho = \frac{1}{1+\cos\theta}$ Parabola $e = 1$

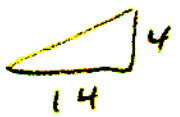
$\rho = \frac{1}{1+0\cos\theta} = 1$ circle $e = 0$

EXTRA

area of rectangles

$2 + 8 + 32 = 42$

Triangle



area $= 28$

$42 - 28 = 14$

Shaded area is 14.

I don't like the 2nd problem – It was phrased poorly then on the change – The answer doesn't seem to make sense. – It Needs more work.

with The change as written on other side – – – –

This is a great, EASY Problem!

100	0	97
90	3	
80	5	
70	7	
60	3	
59	1	55

78.1

Advanced Placement Calculus Name Calc Ch 11

Copy original problem. Per _____ Date 1/19/96

Convince *me* that **you** understand the concept!

Chapter 11 Exam

I The angle of inclination λ of the tangent to a curve at the point $P(r, \theta)$ on it is given by:

$$\tan \lambda = \frac{4 \cos \theta + r' \sin \theta}{-r \sin \theta + r' \cos \theta}$$ where r' means the derivative of r with respect to θ.

Determine all θ which would have a vertical tangent to: $r = 1 + \sin \theta$. (20 pts)

II If the rate at which an ice cube melts in cubic inches per second is directly proportional to the total surface area of the cube with proportionality constant k, Show that a single side of the cube decreases at a constant rate. (20 pts)

and that the area of the edge of a side does not change at a constant rate.

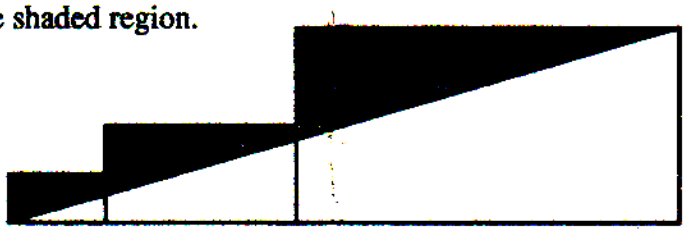
III Given: $2x \sin y + y \cos x - 2y = 0$. Find $\frac{dy}{dx}$ at $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ (20 pts)

IV A ceramist places a vase whose temperature is 120 degrees in a room where the temperature is kept at 70 degrees. According to what is called Newton's law of cooling, the difference between the temperature of the vase and the temperature of the room will decrease exponentially. Suppose that for this vase this difference decreases exponentially at the rate of 25 percent per hour. Write the function which gives the temperature of the vase at any time t. Determine what the temperature of the vase will be 8 hours after the ceramist places it in the room. (hint: To decrease 25% is to retain 75%.) (20 pts)

V Describe the similarities and differences between the Cartesian and the polar graphing planes. Include in your discussion directions for converting from one form to the other and back again. List and identify four polar equations: one for a straight line through the pole, one for a straight line NOT through the pole; one for a circle with center at the pole; one for a circle with center NOT at the pole. List the equivalent Cartesian equation for each polar equation. Briefly describe eccentricity including examples of the equations in polar form of an ellipse, a circle, a hyperbola, and a parabola. (tot 20 pts)

Extra Credit ---------- 5 pts ----------

Three rectangles are lined up horizontally as shown. The first rectangle has width of 1 and length of 2. The second rectangle has a width of 2 and a length of 4. The third rectangle has a width of 4 and a length of 8. Find the area of the shaded region.



I $x^3 + xy^2 + x^3y^5 = 3$

$3x^2 + y^2 + 2xy\frac{dy}{dx} + 3x^2y^5 + 5x^3y^4\frac{dy}{dx} = 0$

(1,1) $3 + 1 + 2\frac{dy}{dx} + 3 + 5\frac{dy}{dx} = 0$

$\rightarrow \frac{dy}{dx} = -1$

$y - y_1 = m(x - x_1)$

$y - 1 = -1(x-1)$

$y - 1 = -x + 1$

$y = -x + 2$

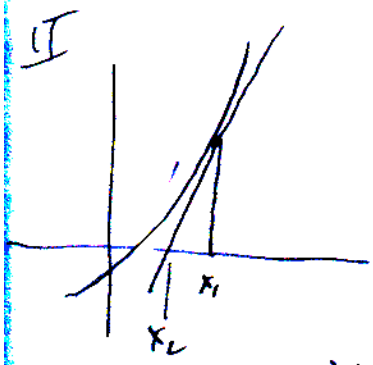
$(-2, 3)$

$3 = 2 + 2$

$= 4$

$3 \neq 4$ No. $(-2,3)$ does not lie on the tangent line to the equation at $(1,1)$.

II



x_1 is 1st guess. x_2 is the x-intercept of the tangent line to $f(x)$ at x_1, when $y=0$ $x = x_2$ is the next guess and so on. You are using the tangent line to approximate the x-intercept of the equation.

$y - f(x_1) = f'(x_1)(x - x_1)$

$-\frac{f(x_1)}{f'(x_1)} = x_2 - x_1$

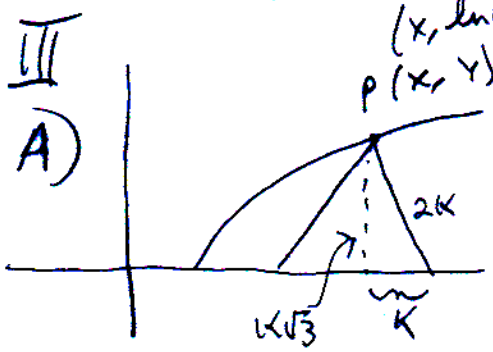
$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

If $f' = 0$ then formula doesn't work because f is a constant and doesn't cross the x-axis or is the x-axis.

when $f' = 0$ and $f' \neq 0$, the tangent line is the f(x) and $\in x \neq$ crossing occurs on 1st guess.

III

A)



$\frac{dx}{dt} = \sqrt{3}$

$\frac{dA}{dt} = ?$

$A = \frac{1}{2}bh$

$= \frac{1}{2} \cdot 2K \cdot K\sqrt{3}$

$= K^2\sqrt{3}$

$A = \frac{\sqrt{3}}{3}(\ln x)^2$

when $\ln x = 1$ $\therefore x = e$

$\ln x = K\sqrt{3}$

$\frac{1}{\sqrt{3}}\ln x = K$

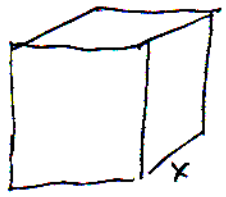
$\frac{dA}{dt} = \frac{2\sqrt{3}}{3}(\ln x) \cdot \frac{1}{x}\frac{dx}{dt}$

$= \frac{2\sqrt{3}}{3} \cdot \ln e \cdot \frac{1}{e} \cdot \sqrt{3}$

$= \frac{2}{e} \approx .736$

The area of the triangle is increasing at the rate of $\frac{2}{e}$ square units per second.

III B)

① 

$\frac{ds}{dt} = -\frac{1}{2}$

$S = 6x^2$

$V = x^3$

$\frac{dV}{dt} = ?$

$\frac{ds}{dt} = 12x \frac{dx}{dt}$

$S = 54$

$\therefore x = 3$

$-\frac{1}{2} = 12x \frac{dx}{dt}$

$-\frac{1}{24x} = \frac{dx}{dt}$

$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$

$= 3x^2\left(-\frac{1}{24x}\right)\Big|_{x=3}$

$\frac{dV}{dt} = -\frac{3}{8}$

the Volume is decreasing at the rate of $\frac{3}{8}$ cu inches/min

② Let $f = \text{face} = x^2$ (area)

$\frac{df}{dt} = ?$

$f = x^2$

$\frac{df}{dt} = 2x \frac{dx}{dt}$

$= 2 \cdot 3 \cdot \left(\frac{-1}{24 \cdot 3}\right)$

$= \frac{1}{12}$ Sq inches/min

Extra

loses $\frac{1}{2}$ sq"/min and has 54 sq" at 3pm

108 minutes later 4:48 pm the cube disappears

IV Polar Pict identical to Cartesian Pict.

Pole = origin pos x axis = polar axis.

$x = r\cos\theta$

$y = r\sin\theta$

$x^2 + y^2 = r^2$

$\tan\theta = \frac{y}{x}$

① $y = x$, $\theta = \frac{\pi}{4}$

② $y = -x$, $\theta = \frac{3\pi}{4}$

③ $y = x + 1$

$r\sin\theta = r\cos\theta + 1$

$r = \frac{1}{\sin\theta - \cos\theta}$

④ $y = -x + 1$

$-r\sin\theta = r\cos\theta + 1$

$r = \frac{-1}{\sin\theta + \cos\theta}$

⑤ $x^2 + y^2 = 1$

$r = 1$

⑥ $(x-1)^2 + y^2 = 1$

$r = 2\cos\theta$

⑦ $(x+1)^2 + (y+1)^2 = 1$

$x^2 + 2x + 1 + y^2 + 2y + 1 = 1$

$x^2 + y^2 + 2x + 2y + 2 = 1$

$r^2 + 2r\cos\theta + 2r\sin\theta = -1$

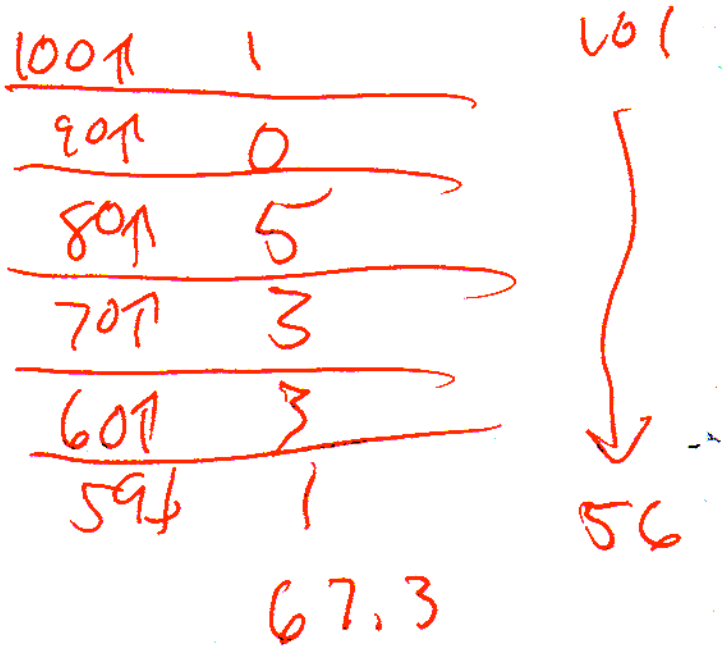
$r = \frac{1}{1 + \frac{1}{2}\cos\theta}$ ellipse $e = \frac{1}{2} < 1$

$r = \frac{1}{1 + \frac{3}{2}\cos\theta}$ hyperbola $e = \frac{3}{2} > 1$

$r = \frac{1}{1 + \cos\theta}$ parabola $e = 1$

$r = \frac{1}{1 + 0\cos\theta}$ circle $e = 0$

eccentricity is the ratio of the distance from a point to the focus over the distance from that point to the directrix.



67.3

Students refused to believe that they needed to use their time wisely. To ensure this I think adding a couple of Quizzes each week might help.

Advanced Placement Calculus

Copy original problem.

Convince *me* that **you** understand the concept!

Name Calc Ch 11

Per _____ Date 99

Chapter 11 Exam

I Show that the tangent line to the curve $x^3 + xy^2 + x^3y^5 = 3$ at the point $(1, 1)$ does or does not pass through the point $(-2, 3)$?

(10 pts)

II *Explain* how "Newton's method" solves equations. Do not select any specific functions. Draw a picture of a generic function. Show graphically what "Newton's method" does. Derive the recursion identity.

Include a discussion of the significance of (1) when $f'(x) = 0$ and

(2) when $f''(x) = 0$ and $f'(x) \neq 0$ (20 pts)

III *Related rates. Draw a picture, identify required items, solve, answer using complete sentences.*

a) A point P with coordinates (x, y) moves along the curve $y = \ln x$ so that its abscissa value (x–coordinate) increases at the rate of $\sqrt{3}$ inches per second. If P is one vertex of an equilateral triangle whose other two verticies are on the x–axis, find out how fast the area of the triangle is changing when the ordinate (y–coordinate) value is one. (25 pts)

b) A cube is contracting so that its surface area decreases at the constant rate of one-half square inch per minute. (25 pts tot)

1. Determine how fast the volume is changing when at 3 P.M. the surface area is fifty-four square inches.

2. Determine how fast the area of one of the faces is changing at the same time.

IV Describe the similarities and differences between the Cartesian and the polar graphing planes. Include in your discussion directions for converting from one form to the other and back again. List, identify and label with the cartesian as well as the polar equations: one for a straight line through the pole with positive slope, one for a straight line through the pole with negative slope, one for a straight line with positive slope NOT through the pole; one for a straight line with negative slope NOT through the pole; one for a circle with center at the pole; one for a circle with center NOT at the pole; one for a circle with center located in what would be called the "second quadrant" if we were using cartesian rather than polar labeling. Briefly describe eccentricity including examples of the equations in polar form of an ellipse, a circle, a hyperbola, and a parabola. (20 pts)

Extra Credit ---------- 5 pts .....

Refering to *III* above: When (if at all) will the cube disappear?

Subject: Re: breaks in domain (was RE: log cabin, song, & competition)
Date: 12/16 5:11 PM
Received: 12/16 8:54 PM
From: Dave L. Renfro, dlrenfro@gateway.net
Reply-To: Post AP Group, ap-calc@ets.org
To: Post AP Group, ap-calc@ets.org

Joshua Zucker <jzucker@pausd.palo-alto.ca.us>
[AP-CALC Wed, 15 Dec 1999 18:44:10 -0800]
<http://forum.swarthmore.edu/epigone/ap-calc/sixflarten>

wrote

>I think this is a very important point and it is one that I have
>trouble communicating. People (including the AP committee, if you
>recall that differential equation in the Acorn book a few years ago)
>just don't check for breaks in the domain when they're solving
>differential equations. Does anybody have good teaching ideas to
>help students develop this habit and understand that there'll be
>different constants of integration (different initial conditions)
>on different parts of the domain?

The following excerpt of a related issue may be of interest
to those interested in these matters.

##
##

From pages 119-120 of Ralph Palmer Agnew, DIFFERENTIAL
EQUATIONS (2'nd edition), McGraw-Hill, 1960.

A careless attempt to find all of the solutions of the equation

(4.52) $(y')^2 = x^2$

consists in asserting that the equation holds if and only
if y'=x or y'=-x and hence that the solutions are given by

(4.53) $y = (1/2)*x^2 + C$ $-\infty < x < \infty$
(4.531) $y = -(1/2)*x^2 + C$ $-\infty < x < \infty$.

Graphs of these solutions are easily sketched. The graphs of
(4.53) form a family of parabolas opening upward, and the
graphs of (4.531) form a family of parabolas opening downward.
Through each point (a,b) of the plane there passes exactly one
parabola of each family.

Let us now solve equation (4.52), paying careful attention to
what we are doing. Assuming that y(x) is a solution of (4.52),
we can transpose and factor to obtain

(4.54) $[y'(x) - x] * [y'(x) + x] = 0.$

It is a fundamental fact of arithmetic that, if the product
of two numbers A and B is 0, then at least one of A and B
must be 0. Hence it is true that, for each x, at least one
of the two factors in (4.54) must be 0; but (4.54) definitely

does not imply that either the first factor is 0 for all values of x or the second factor is 0 for all values of x.

{footnote: An assumption to the contrary is as silly as the assumption that, if two men A and B keep a pump going continuously, then either A pumps all the time or B pumps all the time. All that can be concluded is that B must be pumping at least when A is not pumping.}

All one can conclude from (4.54) is that the first factor must be 0 for some set of values of x and the second must be 0 for all other values of x. Let E (the initial letter of the French word *ensemble* which means *set*) denote the set of x for which the first factor is 0. The nature of the set E is determined by the function y(x); so far as we know at present it may contain no values of x, it may contain all values of x, or it may be any set containing some but not all values of x. Then

```
y'(x) =  x    x in   E
      = -x    x not in E.
```

The character of the set E can be determined by the use of Rolle's theorem as follows: If 0 < a < b and y(a) = y(b), then Rolle's theorem would imply existence of a point c between a and b such that y'(c) = 0; but this is impossible since

```
|y'(x)| = |x| \= 0    [ \= means "not equal"]
```

when a < x < b. Therefore y(a) \= y(b) when 0 < a < b. Since y(x) is continuous, this implies that y(x) must be an increasing function of x for x > 0 or y(x) must be a decreasing function of x for x > 0. In the first case, y'(x) >= 0 so that y'(x) = x for x > 0; and in the second case, y'(x) <= 0 so that y'(x) = -x for x > 0. It can be shown in the same way that y'(x) = x for x < 0 or y'(x) = -x for x < 0. Moreover, (4.52) implies that y'(0) = 0. Therefore there are just four possible cases: (i) y'(x) = x for all x; (ii) y'(x) = -x for all x; (iii) y'(x) = x for x >= 0 and y'(x) = -x for x <= 0; and (iv) y'(x) = -x for x >= 0 and y'(x) = x for x <= 0. {Two typos in case (iv) have been corrected here.} In case (i) we have y(x) = (1/2)*x^2 + C as in (4.53); in case (ii) we have y(x) = -(1/2)*x^2 + C as in (4.531); in case (iii) we have

```
(4.55)    y(x) =  (1/2)*x^2 + C        x >= 0
             = -(1/2)*x^2 + C       x <= 0;
```

and in case (iv) we have

```
(4.551)   y(x) = -(1/2)*x^2 + C        x >= 0
             =  (1/2)*x^2 + C       x <= 0.
```

Thus each solution of (y')^2 = x^2 must have one of the four forms. It is easy to show that each such function satisfies

Given $x^2 + xy + y^2 = 3$. 25 pts

a) find equation of tangent line at $(1,1)$

$$\frac{dy}{dx} = \left.\frac{-(y+2x)}{x+5y}\right|_{(1,1)} = -\frac{3}{6} = -\frac{1}{2}$$

b)

c) Is the graph of the function concave up or down at $(1,1)$?

$$y - 1 = -\frac{1}{2}(x-1)$$

$$y = -\frac{1}{2}x + \frac{3}{2}$$

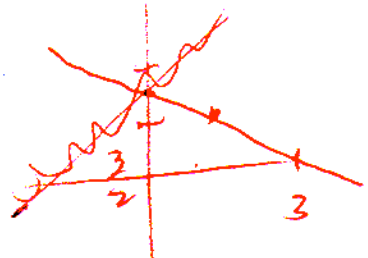
$$\frac{d^2y}{dx^2} = \frac{-(x+5y)\left(\frac{dy}{dx}+2\right) + (y+2x)\left(1+5\frac{dy}{dx}\right)}{(x+5y)^2}$$

$$\frac{-(6)\left(-\frac{1}{2}+2\right) + (1+2)\left(1+\frac{5}{2}\right)}{36}$$

The tangent line at $(1,1)$ ~~forms a~~ and the x axis and the y axis form a triangle. What is the area and Perimeter of that triangle?

$$\frac{-6\left(\frac{3}{2}\right) + 3\left(\frac{-3}{2}\right)}{36} < 0$$

$\therefore$



$\frac{9}{4} + 9 \quad \frac{9+36}{4} \quad \frac{45}{4}$

$\frac{3}{2}\sqrt{5}$

Perimeter $\frac{3}{2}\sqrt{5} + \frac{45}{4}$

area: $\frac{9}{2}$

The tortoise and the hare are having their fabled footrace, each moving along a straight line. The tortoise, moving at a constant rate of 10 ft per minute, is 4 feet from the finish line when the hare wakes up 5001 feet from the finish line and darts off after the tortoise. Let x be the distance from the tortoise to the finish line and suppose the distance y from the hare to the finish line is given by $y = 5001 - 2500\sqrt{4-x}$. $\frac{dx}{dt} = -10$

a) How fast is the hare moving when the tortoise is 3 ft from the finish line?

$$\frac{dy}{dt} = \frac{-2500(-1)}{2\sqrt{4-x}} \frac{dx}{dt}$$

$$= -12500$$

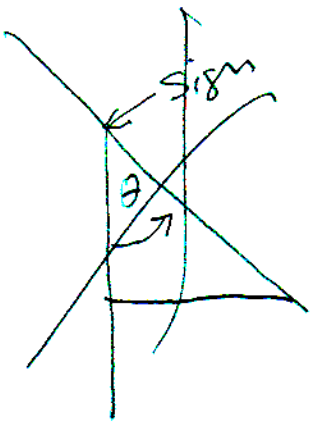
b) When the tortoise crosses the line, $x = 0$ so

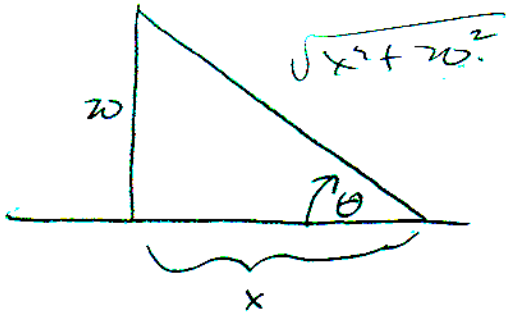
$$y = 5001 - 2500\sqrt{4-0}$$

$$= 5001 - 5000$$

$$= 1$$

So hare is still 1 foot behind.





$\frac{dx}{dt} = -v$ $\qquad \tan\theta = \frac{20}{x}$

$\sec^2\theta \frac{d\theta}{dt} = -\frac{20}{x^2}\frac{dx}{dt}$

$\frac{x^2+400}{x^2}\frac{d\theta}{dt} = -\frac{20}{x^2}(-v)$

$\frac{d\theta}{dt} = \frac{20v}{400+x^2}$

$= \frac{20v}{x^2+400}$

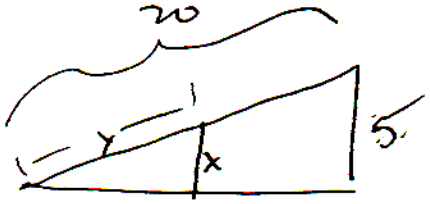
Suppose a deer is standing 20 ft from the highway on which a car is traveling at a constant rate of v feet/sec. Let θ be the angle made by the highway and the line of sight from the car. Show that

$$\frac{d\theta}{dt} = \frac{20v}{400+x^2}$$

Notice that for x close to zero $\frac{d\theta}{dt}$ is approximately $\frac{v}{20}$ and thus for the passenger to keep the deer in focus, the passenger's eyes must rotate at the approximate rate of $\frac{v}{20}$ radians per second. This suggests

at 3 ft/sec

A person is pushing a box up a ramp which is 20 ft long and 5 ft high at it's highest point. How fast is the box ~~rising~~? ~~if the box is being moved at 3 ft/sec~~ ~~rising~~? when the box is 8 ft from the ~~bottom~~ end? when the box is highest?



Both answers are 3/4 ft/sec

$$\frac{x}{y} = \frac{5}{20}$$

$$4x = y$$

$$4\frac{dx}{dt} = \frac{dy}{dt}$$

$$4\frac{dy}{dt} = 3$$

$$\frac{dy}{dt} = \frac{3}{4}$$

(y')^2 = x^2 and the problem is now solved. The natures of the graphs of the solutions in (4.55) and (4.551) should be observed.

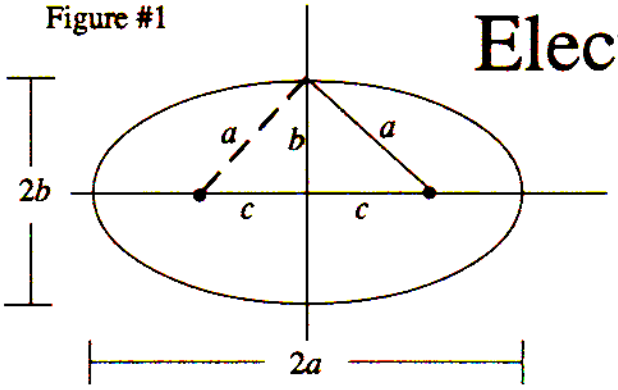
In retrospect, we may remark that the careless solution of (y')^2 = x^2 was very simple but gave incomplete results. The careful solution was a little more difficult, but it gave complete results.

##########
##########

Dave L. Renfro

Electric Eccentricity

Figure #1

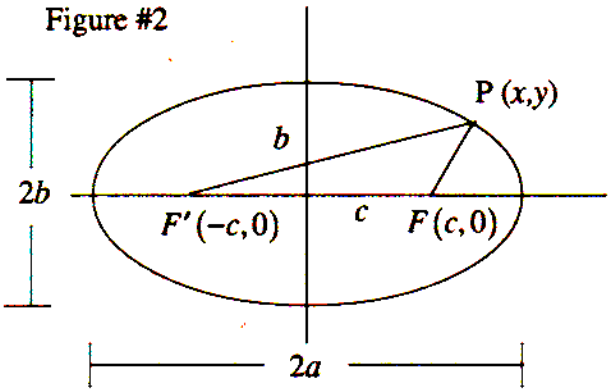


$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

Algebra definition of an ellipse: $PF' + PF = 2a$

$$\sqrt{(x+c)^2+y^2}+\sqrt{(x-c)^2+y^2}=2a$$

Figure #2



Isolate one radical and square

$$\sqrt{(x+c)^2+y^2}=2a-\sqrt{(x-c)^2+y^2}$$

$$(x+c)^2+y^2=4a^2-4a\sqrt{(x-c)^2+y^2}+(x-c)^2+y^2$$

$$x^2+2cx+c^2+y^2=4a^2-4a\sqrt{(x-c)^2+y^2}+x^2-2cx+c^2+y^2$$

$$4cx=4a^2-4a\sqrt{(x-c)^2+y^2}$$

$$cx=a^2-a\sqrt{(x-c)^2+y^2}$$

$$a^2-cx=a\sqrt{(x-c)^2+y^2}$$

$$a^2-cx=a\underbrace{\sqrt{(x-c)^2+y^2}}_{PF}$$

Solve for PF

$$PF=a-\frac{c}{a}x$$

This ratio is our main concern. Notice that $c < a$ because (see figure #1) the leg, c of the right triangle must be less than the hypotenuse, a so our ratio is less than 1 but greater than zero.

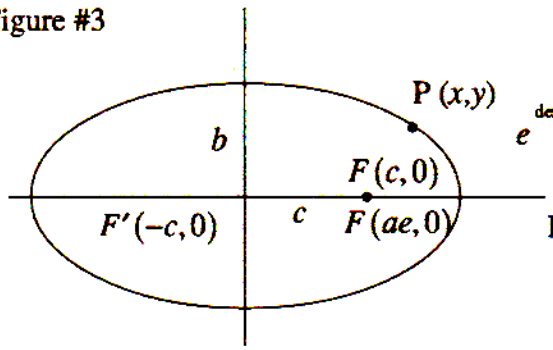
We shall call that ratio e.
e stands for the word *eccentricity* which refers to "how curvey" a line is *and not for the base of the natural logs.* You usually hear the word eccentric in reference to a person who acts differently than the norm. They are said to be "slightly off center." In our application the foci are "slightly off the center."

Thus, $PF = a - e\,x$

factor e

$$PF=e\left(\frac{a}{e}-x\right)$$

Figure #3

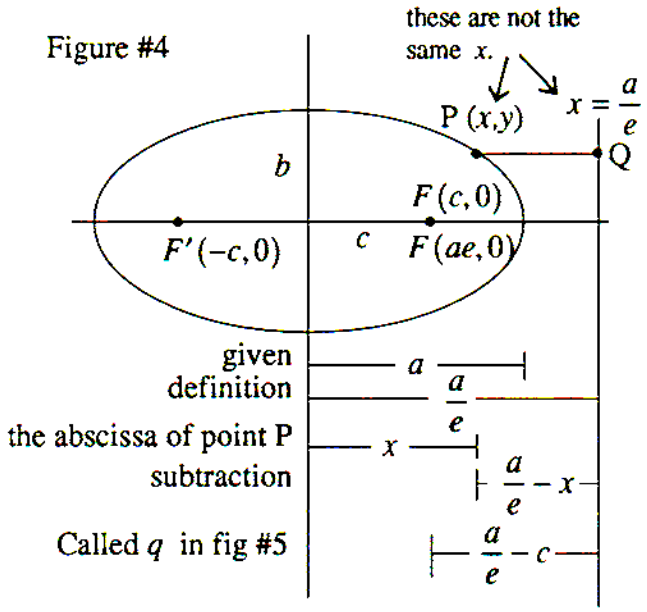


$e \overset{\text{defined}}{=} \frac{c}{a}$ so $c = ae$

Notice: $c < a$ so

$e < 1$ but positive.

Figure #4



What can we say about this equation? $PF = e\left(\frac{a}{e} - x\right)$

We know e is less than 1 so $\frac{a}{e} > a$ because division by a number smaller than 1 yields a larger quotient (Thus, $\frac{a}{e}$ is beyond the right edge of the ellipse which is a from the center. Let's define the vertical line (parallel to the y-axis) drawn through $\frac{a}{e}$ to be the directrix $x = \frac{a}{e}$. Now, $PF = e\left(\frac{a}{e} - x\right)$ becomes $PF = ePQ$ (where PQ is the distance from P to the directrix.) which leads to:

$$e = \frac{PF}{PQ} = \frac{\text{distance from focus to P}}{\text{distance from P to directrix}} = \frac{c}{a}$$

Figure #5

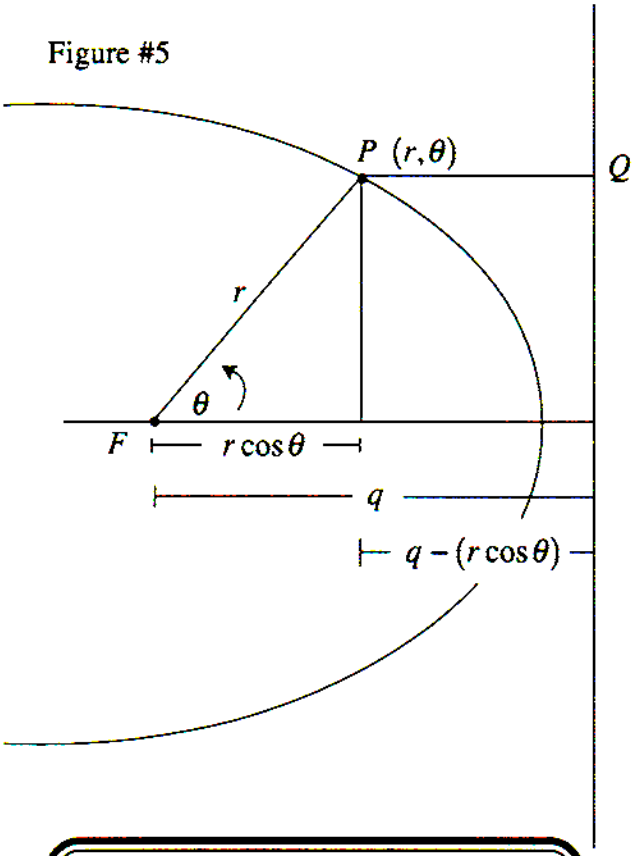


Figure #5 is a blow-up of a portion of figure #4.

$$e = \frac{PF}{PQ} = \frac{r}{q - (r\cos\theta)}$$

$$eq - er\cos\theta = r$$

$$eq = r + er\cos\theta$$

$$r(1 + e\cos\theta) = eq$$

$$r = \frac{eq}{1 + e\cos\theta}$$

Depending on the orientation of the conic, one of the following is the actual "general case" ***if and only if a focus is located at the pole.***

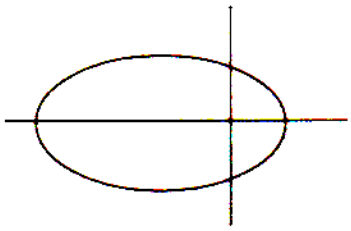
$$r = \frac{eq}{1 \pm e\cos\theta} \quad \text{or} \quad r = \frac{eq}{1 \pm e\sin\theta}$$

"Orientation" refers to the relative location of the foci. In the illustrated example, the right-hand focus was at the pole *and* the major axis was horizontal

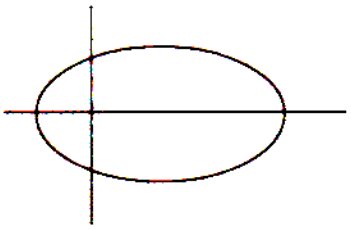
However, regardless of orientation, the following is true:

If $e = 0$ then the conic is a circle.
If $e < 1$ then the conic is an ellipse.
If $e = 1$ then the conic is a parabola.
If $e > 1$ then the conic is a hyperbola.

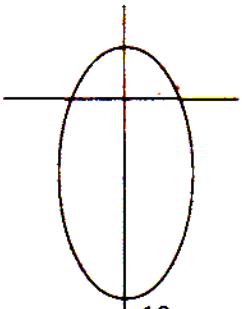
Electric Eccentricity



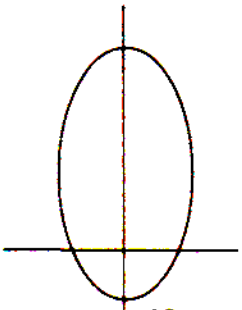
$$r = \frac{10}{3 + 2\cos\theta}$$



$$r = \frac{10}{3 - 2\cos\theta}$$



$$r = \frac{10}{3 + 2\sin\theta}$$



$$r = \frac{10}{3 - 2\sin\theta}$$

See figure # 6 below.

Sketch : $r = \dfrac{10}{3 + 2\cos\theta}$

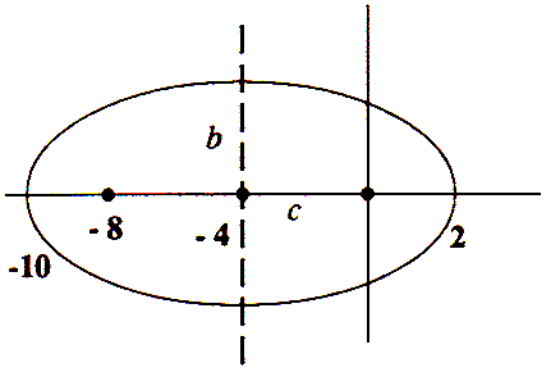
When $\theta = 0$ $r = \dfrac{10}{5} = 2$ and when $\theta = 180$ $r = \dfrac{10}{3-2} = 10$. 2 along the positive x-axis and 10 (actually –10) along the negative x-axis. These are end points of the figure. The line has a length of 2 –(–10)=12.

When $\theta \in \{90,\ 270\}$ $r = \dfrac{10}{3}$. Remember, this is vertical distance *from the pole*!

$$r = \frac{\frac{10}{3}}{\frac{3}{3} + \frac{2}{3}\cos\theta}$$

Because of the $\frac{2}{3}$, we know we are working with an ellipse. The major axis is horizontal and the minor axis is vertical (because we have cosine instead of sine in our equation). The foci are located on the major axis. "a" is equal to one-half of the length of the major axis, so $a = 6$. The center of the ellipse: $2 - 6 = -4$. (4 on the negative x-axis).

Figure #6



$\dfrac{c}{a} = e$ which becomes $\dfrac{c}{6} = \frac{2}{3}$

So, $c = 4$ and $a^2 = b^2 + c^2$ becomes $6^2 = b^2 + 4^2$; thus $b = 2\sqrt{5}$

from figure #1 on the first page

$$r = \frac{\frac{2}{3} \cdot 5}{\frac{3}{3} + \frac{2}{3}\cos\theta}$$

5 is the distance from the a focus to its directrix. So right hand directrix is $x = 5$ and the left hand directrix is $x = -13$ (5 left of left focus).

$$r = \frac{10}{3 + 2\cos\theta}$$

$$3r + 2r\cos\theta = 10$$

$$3\sqrt{x^2 + y^2} + 2x = 10$$

$$3\sqrt{x^2 + y^2} = 10 - 2x$$

$$9x^2 + 9y^2 = 100 - 40x + 4x^2$$

$$5x^2 + 40x + 9y^2 = 100$$

$$5\left(x^2 + 8x + 16\right) + 9y^2 = 100 + 80$$

$$5(x+4)^2 + 9y^2 = 180$$

$$\frac{(x+4)^2}{36} + \frac{y^2}{20} = 1$$

$a = 6$ $\quad b = 2\sqrt{5}$

Electric Eccentricity

Convert to polar notation:

$$\frac{(x \oplus 4)^2}{36} + \frac{y^2}{20} = 1$$

These signs agree → $r = \frac{\quad}{\oplus \cos}$

1) Given is graphing form of an ellipse —> + and equals 1.

2) Major axis horizontal $(36 > 20)$ so we will have cosine in our equation instead of sine

3) $a = 6 \quad b = 2\sqrt{5} \quad \therefore \quad c = \sqrt{36 - 20} = 4$

4) $e = \frac{c}{a} = \frac{4}{6} = \frac{2}{3}$ ⟶ $r = \frac{\frac{2}{3}\ ?}{1 + \frac{2}{3}\cos\theta}$

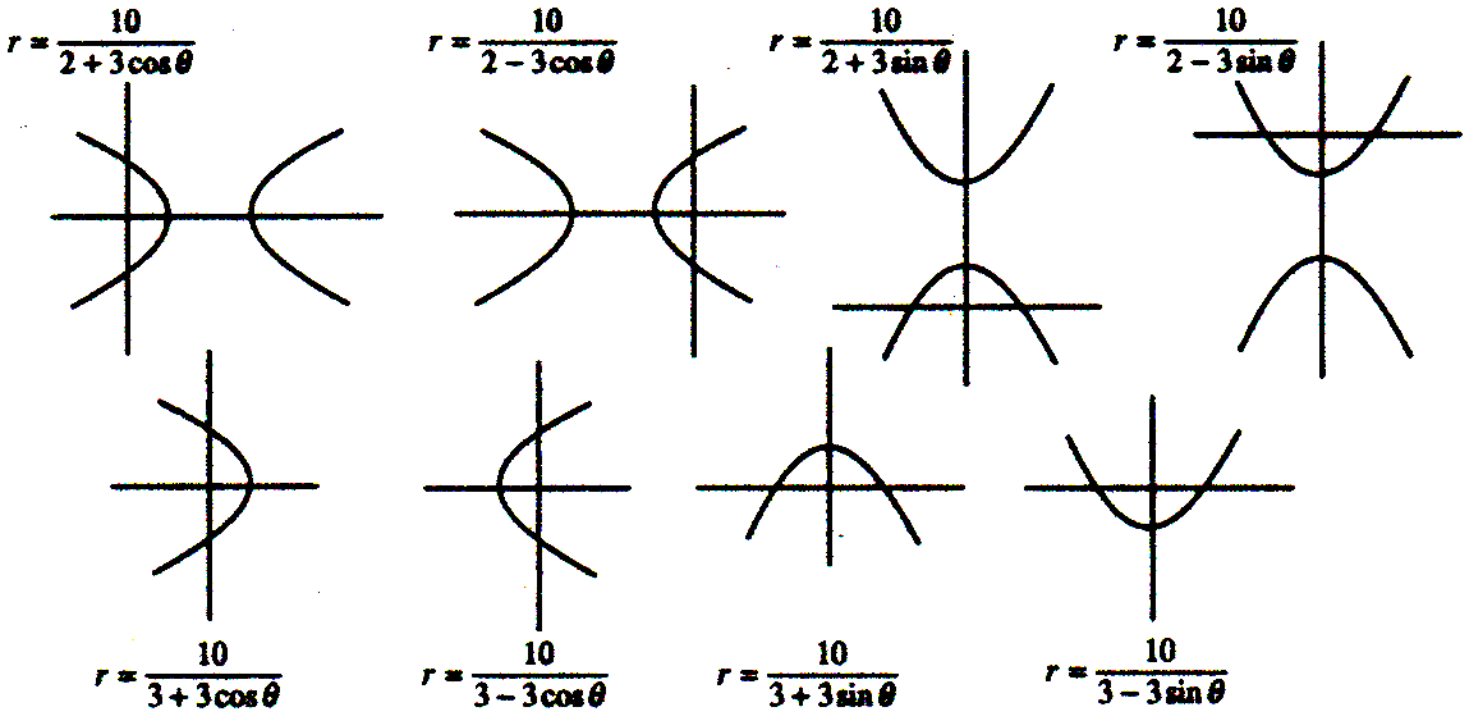
5) The question mark is the distance from the focus to the directrix. See figure # 4

6) The distance from the center of the ellipse to the directrix is: $\frac{a}{e} = \frac{a}{\frac{c}{a}} = \frac{a^2}{c} = \frac{36}{4} = 9.$

So, from $x = -4$ to the right directrix is $-4 + 9 = 5$ and $-4 - 9 = -13$. That is 13 to left directrix.

7) See fig #4, $\frac{a}{e} - c = \frac{6}{\frac{2}{3}} - 4 = 6 \cdot \frac{3}{2} - 4 = 9 - 4 = \textcircled{5}$

$r = \frac{\frac{2}{3} \cdot \textcircled{5}}{1 + \frac{2}{3}\cos\theta}$ $\qquad r = \frac{\frac{2}{3} \cdot 5}{1 + \frac{2}{3}\cos\theta} \cdot \frac{3}{3}$ $\qquad r = \frac{10}{3 + 2\cos\theta}$


$r = \frac{10}{2 + 3\cos\theta}$
$r = \frac{10}{2 - 3\cos\theta}$
$r = \frac{10}{2 + 3\sin\theta}$
$r = \frac{10}{2 - 3\sin\theta}$
$r = \frac{10}{3 + 3\cos\theta}$
$r = \frac{10}{3 - 3\cos\theta}$
$r = \frac{10}{3 + 3\sin\theta}$
$r = \frac{10}{3 - 3\sin\theta}$

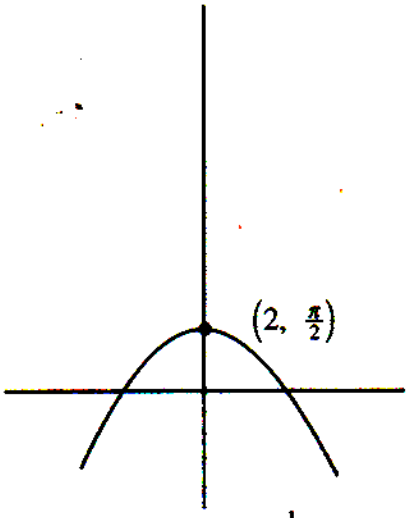
Electric Eccentricity

$$r = \frac{10}{2 + 3\sin\theta} = \frac{5}{1 + \frac{3}{2}\sin\theta}$$

$e = \frac{3}{2} > 1 \quad \therefore$ Hyperbola

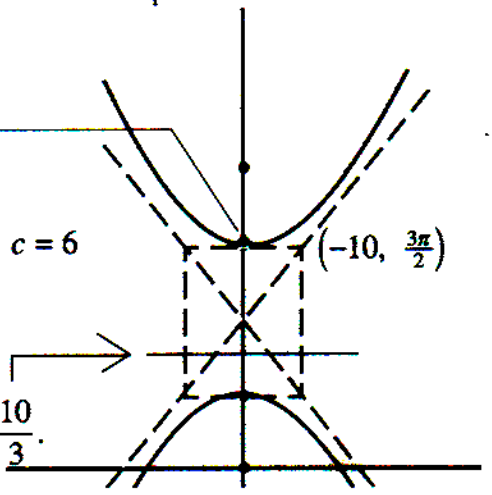
Transverse axis (the one containing the foci) is vertical (because of sine).
The focus is at the lower pole because the equation is in the form:

$$r = \frac{eq}{1 + e\sin\theta}$$



When $\theta = \frac{\pi}{2} \longrightarrow r = 2$

When $\theta = \frac{3\pi}{2} \longrightarrow r = -10$



The length of the transverse axis is $10 - 2 = 8$ so $a = 4$ and $e = \frac{3}{2} = \frac{c}{a} = \frac{c}{4} \quad \therefore \quad c = 6$

$b^2 = c^2 - a^2 = 36 - 16 = 20 \quad \therefore \quad b = 2\sqrt{5}$

$r = \frac{eq}{1 + e\sin\theta} = \frac{5}{1 + \frac{3}{2}\sin\theta} = \frac{\frac{3}{2}\cdot\frac{10}{3}}{1 + \frac{3}{2}\sin\theta}$ so distance from focus to its directrix is $\frac{10}{3}$.

The distance from the "center" to the directrix is $\frac{a}{e}$ exactly as it is in the ellipse however the distance from the focus to the directrix is $q = c - \frac{a}{e}$ The negative of what it is for the ellipse. This is accomplished because e is greater than 1 in a hyperbola

---

I Identify, sketch ,and find the rectangular equivalent and graph of the given equation. Specifically state the equation(s), etc. of asymptotes, directrix, foci, vertex, etc.

a) $r = \frac{6}{3 + \sin\theta}$ b) $r = \frac{6}{1 - 3\sin\theta}$ c) $r = \frac{3}{2 + 2\cos\theta}$

II Express the given equation in polar form and then find the eccentricity and an equation for the directrix:

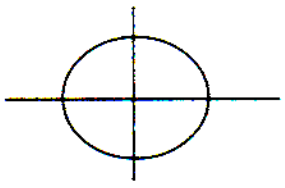
a) $y^2 = 4 - 4x$ b) $3y^2 - x^2 - 12y + 9 = 0$

c) $3x^2 + 4y^2 + 4x - 4 = 0$ d) make up one of your own

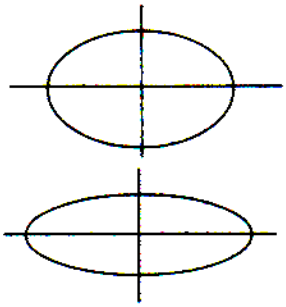
III Match each ellipse with its equation:

a) $r = \frac{10}{5 + 2\cos\theta}$ b) $r = \frac{10}{5 + 3\cos\theta}$

c) $r = \frac{10}{5 + 4\cos\theta}$ d) conclusion ...



2)



1)

3)

The Twelve Days of Christmas

Each of the following homework assignments are to be prepared during the Christmas vacation. You will have to write dialogs of class lectures, including example problems for two topics. As most students have said on more than one occasion, "The book just doesn't present the material in an understandable manner." Here is your chance to show that *you can do* a better job. *Do honors level work!*

Day One ***Day Two***

Write a complete lecture concerning everything there is to know about the topics on pages 183 - 189. You are allowed to use the same examples as those which are in the book however, you are expected to clearly explain their significance in your own words.

Day Three

Demonstrate your understanding of the topics from Days One and Two by doing the following problems: Page 189 1 - 6, 8, 11, 18, 19.

Day Four

Write "narratives" covering every step of each problem in the problem set from Day Three.

Day Five

Do problems: Page 458 1, 2, 5, 7, 10

Day Six ***Day Seven***

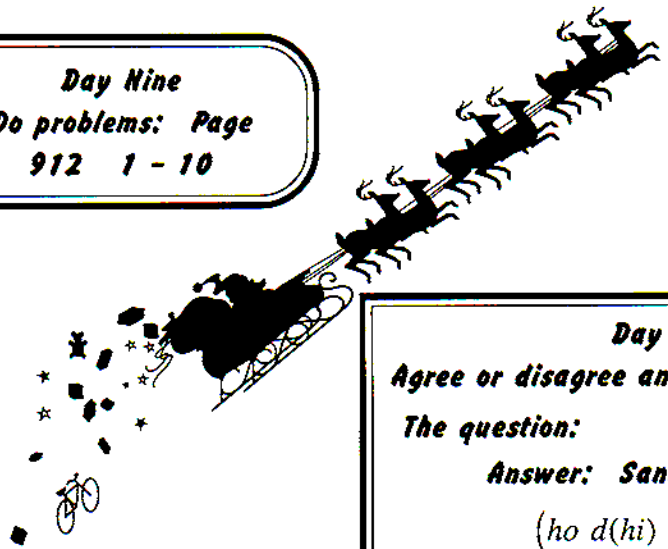
Write a lecture covering the topics on pages 392 - 395. Tell everything there is to know about the topics! Use good examples.

Day Eight

Do Page 396 1 - 17 odd, 21 - 31 odd

Day Nine

Do problems: Page 912 1 - 10



Day Ten

Do Page 914 - 915 1 - 10

Day Eleven

Do problems: Page 920 - 921 1 - 11

Day Twelve

Agree or disagree and justify, of course:

The question: ***What is*** $d(hi \,/\, ho)$***?***

Answer: Santa Says:

$$\frac{(ho\ d(hi) \ - \ hi\ d(ho))}{(ho\ ho)}$$